

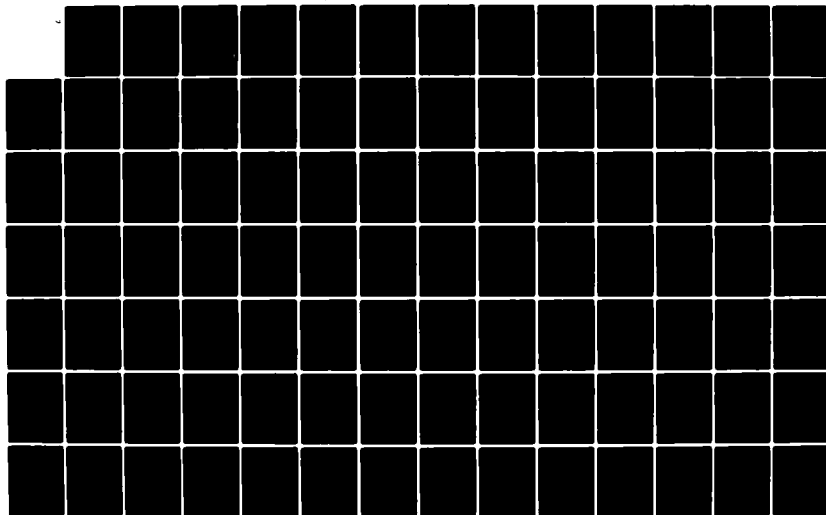
AD-A137 814

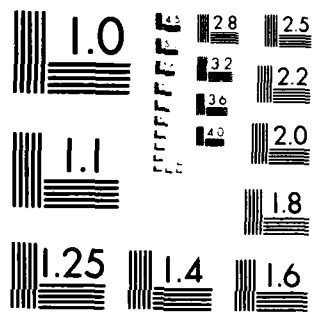
DESIGN OF LONGITUDINAL CONTROL LAWS FOR THE X-29A  
BACKUP ANALOG FLIGHT CO..(U) AIR FORCE INST OF TECH  
WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI... H L EMRICK  
SEP 83 AFIT/GAE/AA/83S-3 F/G 1/3

1/2

UNCLASSIFIED

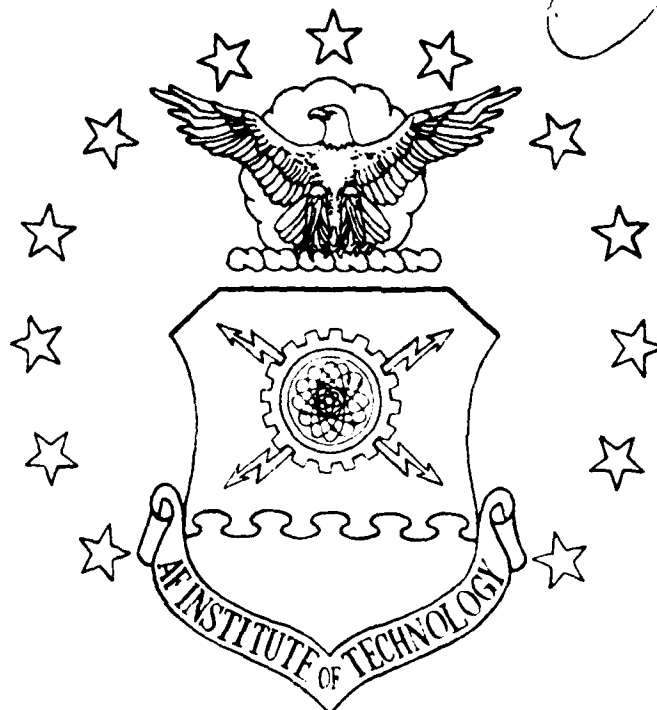
NL





MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

AD A137814



DESIGN OF LONGITUDINAL CONTROL LAWS  
FOR THE X-29A BACKUP ANALOG FLIGHT  
CONTROL SYSTEM

THESIS

AFIT/GAE/AA/83S-3 Holly L. Emrick  
1st Lt USAF

UNITED STATES AIR FORCE  
AIR UNIVERSITY  
AIR FORCE INSTITUTE OF TECHNOLOGY  
Wright-Patterson Air Force Base, Ohio

RECEIVED  
FEB 14 1984  
A

DTIC FILE COPY

This document has been approved  
for public release and its  
distribution is unlimited

AFIT/GAE/AA/83S-3

DESIGN OF LONGITUDINAL CONTROL LAWS  
FOR THE X-29A BACKUP ANALOG FLIGHT  
CONTROL SYSTEM

THESIS

AFIT/GAE/AA/83S-3    Holly L. Emrick  
                         1st Lt        USAF

Approved for public release; distribution unlimited

AFIT/GAE/AA/83S-3

DESIGN OF LONGITUDINAL CONTROL LAWS FOR THE  
X-29A BACKUP ANALOG FLIGHT CONTROL SYSTEM

THESIS

Presented to the Faculty of the School of Engineering  
of the Air Force Institute of Technology  
Air University  
in Partial Fulfillment of the  
Requirements for the Degree of  
Master of Science

by  
Holly L. Emrick, B.S.AE  
1st Lt USAF  
Graduate Aeronautical Engineering  
September 1983

Approved for public release; distribution unlimited

## Preface

The purpose of this study was to design longitudinal control laws for the analog backup flight control system on the X-29A aircraft. The 35% static instability encountered in this aircraft suggested a unique problem.

A stable control law design which meets Level I flying qualities was made with a rigid body and linear equations of motions assumed. The Appendices include computer output from TTYLON and TOTAL, the computer programs used to find transfer functions and do root locus design, respectively.

I would like to thank my advisor, Dr. Robert Calico for his guidance which was essential to the completion of this study. I would also like to thank Lt Col Michael Smith and Dr. Constantine Houppis for their assistance. Gratitude is expressed to Capt Stanley Fuller, Mr. Stan Lash, Lt Kris Farry, Capt Dave Potts, and Mr. Stan Larimer of the Flight Dynamics Laboratory for their advice and help in collecting data. Finally, I wish to acknowledge my gratitude to a special friend, Capt John Koelling for all his advice and encouragement throughout the project.

## Table of Contents

	<u>Page</u>
Preface . . . . .	ii
List of Figures . . . . .	v
List of Tables . . . . .	vi
List of Symbols . . . . .	vii
Abstract . . . . .	viii
I. Introduction . . . . .	1
Problem Statement . . . . .	1
Background . . . . .	1
General Approach . . . . .	2
Sequence of Presentation . . . . .	2
Assumptions . . . . .	2
II. Theory and Development . . . . .	4
Deriving the Transfer Functions . . . . .	4
Root Locus Design . . . . .	8
Choosing Outputs to Feed Back . . . . .	10
Placement of the Normal Acceleration Sensor . . . . .	11
Gain Scheduling . . . . .	16
Multiloop Closure . . . . .	18
III. Design . . . . .	21
Multiloop Design . . . . .	21
Selection of Gains . . . . .	22
Phugoid Design Modification . . . . .	33
IV. Conclusions and Recommendations . . . . .	41
Bibliography . . . . .	42
Appendix A: Wind Tunnel Data . . . . .	44
Appendix B: TTYLON Flow Diagrams . . . . .	50
Appendix C: TTYLON Runs for N <sub>2</sub> Sensor Location . . . . .	58
Appendix D: TTYLON Runs for Flight Envelope . . . . .	61

	<u>Page</u>
Appendix E: Root Loci . . . . .	84
Appendix F: Least Squares Curve Fitting . . . . .	133
VITA . . . . .	136



# List of Figures

<u>Figure</u>		<u>Page</u>
1	X-29A Control Surfaces . . . . .	5
2	Bare Airframe $N_z/\delta_c$ . . . . .	9
3	$N_z/\delta_c$ with Actuator Dynamics . . . . .	9
4	$\alpha/\delta_c$ at $M=0.6$ , $h=20K$ ft . . . . .	12
5	$N_z/\delta_c$ at $M=0.6$ , $h=20K$ ft . . . . .	13
6	Closed $N_z$ , Open $q$ at $M=0.6$ , $h=20K$ ft . . . . .	14
7	Block Diagram of the X-29A Longitudinal Axis . . . . .	19
8	$N_z/\delta_c$ at $M=0.5$ , $h=40K$ ft . . . . .	24
9	$N_z/\delta_c$ at $M=0.8$ , Sealevel . . . . .	24
10	$K_q$ vs Dynamic Pressure Curve Fitting . . . . .	27
11	Short Period Mode Flying Qualities Requirements . . . . .	32
12	$\omega_{sp}T_{\theta_2}$ vs $\zeta_{sp}$ for Design Results . . . . .	34
13	Time Response for $q/\delta_c$ at $M=0.3$ , Sealevel . . . . .	36
14	Time Response for $q/\delta_c$ with Lag Compensation at $M=0.3$ , Sealevel . . . . .	36
15	Closed $N_z$ , Open $q$ Loop Phugoid Without Filter at $M=0.3$ , Sealevel . . . . .	40
16	Closed $N_z$ , Open $q$ Loop Phugoid With Filter at $M=0.3$ , Sealevel . . . . .	40

### List of Symbols

$E(s)$	Error Signal
$g$	Gravitational Acceleration (ft/sec <sup>2</sup> )
$G_c(s)$	Command Lag Transfer Function
$G_f(s)$	Lag Filter Transfer Function
$G_s(s)$	Servo Transfer Function
$h$	Altitude (ft)
$K_{N_z}$	$N_z$ Feedback Gain
$K_q$	$q$ Feedback Gain
$L_x$	Distance from $N_z$ Sensor to Center of Gravity (ft)
$M$	Mach Number
$N_z$	Normal Acceleration (ft/sec <sup>2</sup> )
$Q$	Dynamic Pressure (lb/ft <sup>2</sup> )
$q$	Pitch Rate (rad/sec)
$T_{\theta_2}$	Period of Higher Frequency $\theta/\delta_c$ Zero (sec)
$u$	Perturbation Velocity in the x Direction (ft/sec)
$U_o$	Free Stream Velocity in the x Direction (ft/sec)
$\alpha$	Angle of Attack (rad)
$\delta_c$	Canard Deflection (rad)
$\delta_p$	Pilot Command (rad)
$\zeta_{sp}$	Short Period Damping Ratio
$\theta$	Pitch Angle (rad)
$\theta_o$	Initial Value Pitch Angle (rad)
$\omega_{sp}$	Short Period Frequency (rad/sec)

### Abstract

X-29A wind tunnel data were input to the computer program TTYLON to produce  $\alpha/\delta_c$ ,  $N_z/\delta_c$ , and  $q/\delta_c$  bare airframe transfer functions throughout the flight envelope. The short period mode below Mach 1 was unstable. Where it was stable, the response was slow.

The normal acceleration sensor was placed so that the short period frequency would be within the desired range of 3 to 10 rad/sec.  $N_z$  feedback was used to stabilize the short period mode. Pitch rate feedback was also used to add damping for Level I flying qualities in the short period mode. To stabilize the phugoid mode and achieve Level I flying qualities, a lag filter design was developed. Gain schedules for  $K_{N_z}$  and  $K_q$ , which were found using classical multiloop control law design are presented.

## I. Introduction

### Problem Statement

The purpose of this thesis is to use classical control techniques to design backup analog control laws for the X-29A Forward Swept Wing aircraft. The 35% negative static margin inherent in the airframe makes this a unique problem. The design and analysis in this study are limited to the longitudinal axis. Therefore, all references to control laws and system response will include only the longitudinal modes.

### Background

Wing sweep was introduced to aircraft when it was discovered that the critical Mach number could be increased, thus delaying the transonic drag rise. Sweepback, however has some disadvantages, such as: tip stall, which reduces controllability; poor lateral control; and increased induced drag at high incidence angles. Forward sweep while still reducing critical Mach number, promises to reduce trim drag on the aircraft and increase maneuverability since stall begins at the root of the wing.

The canard-forward swept wing configuration being used by Grumman has the above mentioned 35% negative static margin for subsonic flight. It becomes stable in supersonic flight. A triplex digital flight control system with an analog backup will be used to handle the instability. This allows the aircraft to be operational after a failure in one or all of

the digital flight control computers. Modern control theory, specifically implicit model following is being used as the main digital control law design tool by Grumman. In this study, as with the original design, classical frequency domain methods will be used for the analog backup control law design.

### General Approach

The aircraft aerodynamic characteristics found in wind tunnel testing were used in the computer program TTYLON to determine the aircraft longitudinal axis transfer functions. The computer program TOTAL was then used to do frequency domain analysis and design. Feedback states were decided upon and gains and/or gain schedules calculated for the final system design.

### Sequence of Presentation

The theoretical basis for the transfer function calculations, root locus design and analysis, multiloop closure, and gain scheduling are presented in Chapter II. The actual steps taken in design can be found in Chapter III. Chapter IV contains the conclusions drawn and recommendations made for the study.

### Assumptions

The longitudinal equations of motion for the aircraft are assumed to be the linear perturbation equations about

steady level flight. The aircraft is also assumed to be a rigid body. These assumptions must be used to allow use of existing tools and to limit the scope of the problem.

## II. Theory and Development

### Deriving the Transfer Functions

Wind tunnel data were obtained from the X-29A Advanced Development Program Office (ADPO). These data are in the form of stability derivatives, lift and drag coefficients, weights and moments of inertia. The data may be found in Appendix A. The computer program TTYLON - A Longitudinal Aircraft Transfer Function Program, which is maintained by ASD/ENFTC at Wright-Patterson AFB, was used to calculate transfer functions from the wind tunnel data for selected aircraft configurations and flight conditions. The flow diagram for the program is shown in Appendix B and the dotted lines indicate the options which were used. The stability derivatives were found for a trimmed aircraft at the desired flight condition. The longitudinal perturbation state equations then were solved for the transfer functions. The equations used are:

$$\begin{bmatrix} (1-X_u^*)s-X_u & -X_u^*s-X_x & W_0-X_q & g \cos \theta_0 \\ -Z_u^*s-Z_u & (u_0-Z_x^*)s-Z_x & -U_0-Z_q & g \sin \theta_0 \\ -M_u^*s-M_u & -M_x^*s-M_x & s-M_q & 0 \\ 0 & 0 & -1 & s \end{bmatrix} \begin{bmatrix} u \\ \alpha \\ q \\ \dot{\theta} \end{bmatrix} =$$

$$\begin{bmatrix} X_{\delta_e} & \frac{\cos \epsilon_T}{M} \\ Z_{\delta_e} & \frac{-\sin \epsilon_T}{M} \\ M_{\delta_e} & \frac{Z_T \cos \epsilon_T}{I_{yy}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \Delta T \end{bmatrix} \quad (1)$$

(9:19)

$N_z$ , the normal acceleration, is given by the following equation:

$$N_z = U_0 q/g + L_x q/g - U_0 \alpha/g - \dot{\theta} \sin \theta_0 \quad (2)$$

As can be seen in Fig 1, the X-29A uses three types of surfaces for longitudinal trim and control.

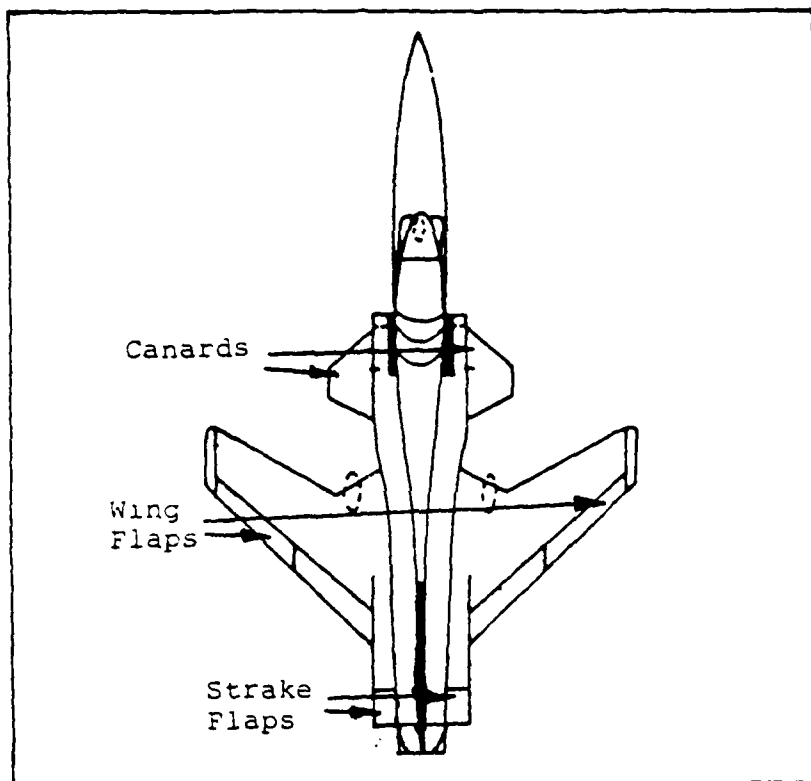


Fig 1. X-29A Control Surfaces

The canard is moved to the position necessary for the proper pitching moment, then the wing flaps and strake flaps are scheduled to unload the canard so that it may assume the position for minimum trim drag at that flight condition. In this study, only the canard was considered as the control



input. This assumption is valid because it is the only active control surface; the others are passive. Whenever an elevator input is referred to in TTYLON, the canard should be substituted.

The flight conditions are defined by Mach, altitude, weight, center of gravity, and flight path angle in TTYLON. For this study, the following were held constant:

- 1) Weight = 15,000 lb
- 2) Center of Gravity = 451 in (37.58 ft)
- 3) Flight Path Angle = 0 degrees

The flight conditions chosen for control law design are shown in Table I.

Table I  
Flight Conditions Used for Control System Design

Altitude (ft)	Mach
0	0.3
	0.6
	0.8
20K	0.4
	0.6
	1.0
	1.2
40K	0.5
	0.6
	1.0
	1.4

This flight envelope is limited at low Mach by aerodynamics (stall), and at high Mach by structural limits (dynamic pressure). The maximum dynamic pressure is 1200 lb/ft<sup>2</sup>. The roots of the bare airframe canard transfer functions at the various flight conditions are shown in Table II.

Table II  
Characteristic Roots of Bare Airframe  
Canard Transfer Functions

Flight Condition		Short Period		Phugoid	
M	h(ft)	real	imag	real	imag
0.3	0	1.9478	0	-0.14311e-1	0.14132
		-3.0539	0		
0.6	0	4.0251	0	-0.10817e-1	0.69910e-1
		-6.6276	0		
0.8	0	5.4774	0	-0.16487e-1	0.41562e-1
		-9.3031	0		
0.4	20K	1.8793	0	-0.12824e-1	0.11286
		-1.9912	0		
0.6	20K	2.8675	0	-0.70548e-2	0.76164e-1
		-4.0591	0		
1.0	20K	3.9644	0	-0.39750e-1	0
		-6.0004	0	0.16974e-1	0
1.2	20K	3.4147	0	-0.91117e-1	0
		-5.7362	0	0.53264e-1	0
0.5	40K	1.5780	0	-0.15728e-1	0.94651e-1
		-1.9912	0		
0.6	40K	1.9091	0	-0.10247e-1	0.80888e-1
		-2.4136	0		
1.0	40K	2.6941	0	-0.67527e-2	0.47972e-1
		-3.5394	0		
1.4	40K	1.5136	0	-0.82773e-2	0.39491e-1
		-2.5960	0		

Once the bare airframe transfer functions were found, a test was made to find what effect the canard actuator dynamics had upon the bare airframe dynamics. The transfer function for the canard actuator is given by:

$$G_a(s) = \frac{(20.202)(144.928)(71.4)^2}{(s+20.202)(s+144.928)(s+52.55+j48.34)} \quad (3)$$

Bode plots of the open loop  $N_z/\delta_c$  transfer function with and without the actuator included are shown in Figs 2 and 3. One can see that the response is affected very little and only above 40 rad/sec which is not in the frequency range of the longitudinal mode dynamics. The effect the actuator had on dynamics was therefore considered negligible and the transfer function was assumed to be 1.

The sensor and pilot command transfer functions were also assumed to have little effect on the aircraft response. This is because the lag in both cases is, in general, an order of magnitude smaller than the period of the short period mode.

#### Root Locus Design

The computer program TOTAL, which is maintained by Mr. John Smith, ASD/ENFTC, was the main tool in this design and analysis of the control laws. Once the bare airframe transfer functions were found using TTYLON, they were put in TOTAL. Option 41 produces a listing of the root locus.

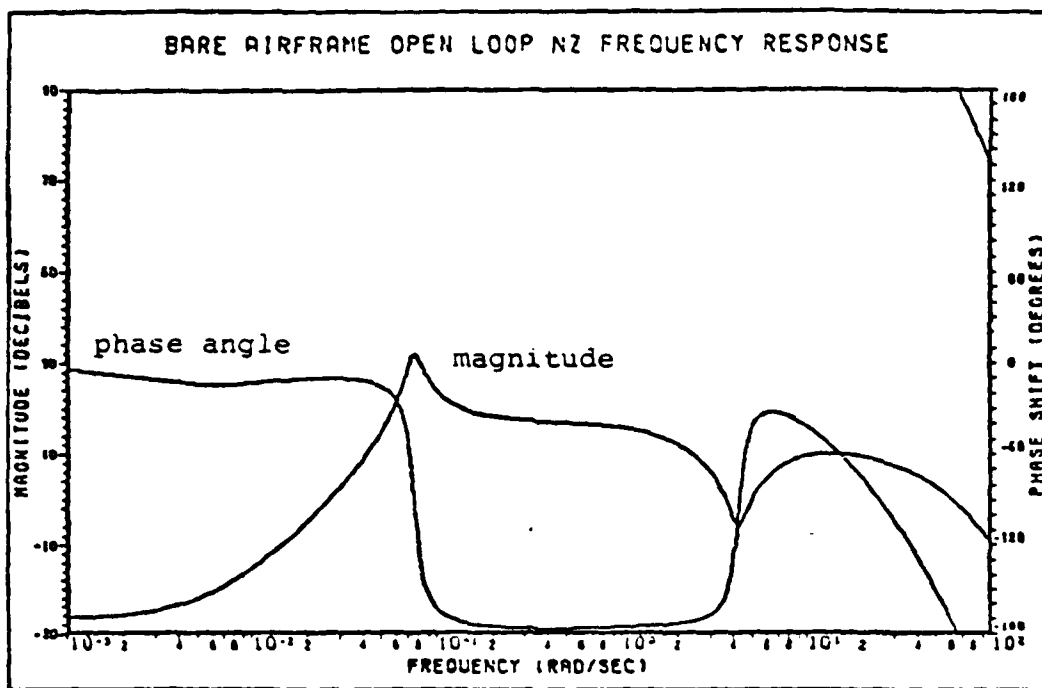


Fig 2. Bare Airframe  $N_z/\delta_c$

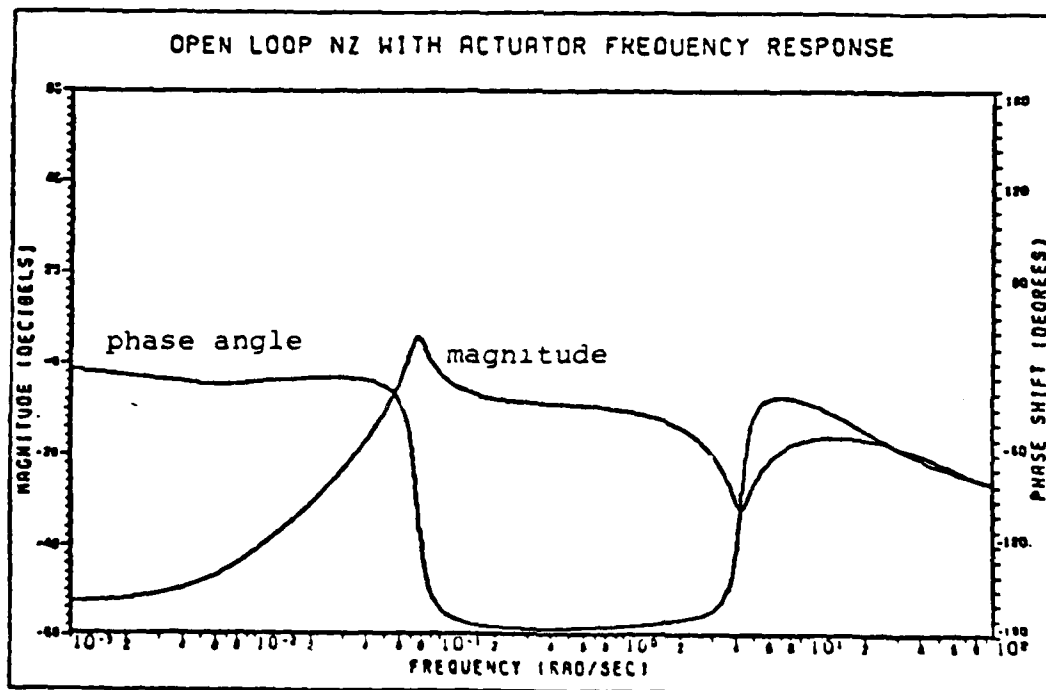


Fig 3.  $N_z/\delta_c$  with Actuator Dynamics

Looking at an example in Appendix E, one can see that the gain and damping ratio at each point on the locus is listed. By looking at these listings for a series of flight conditions, one may find the gain at which damping ratio, damped frequency, or stability criteria may be met. Option 42 in TOTAL allows the designer to obtain the closed loop roots for the system with a specified gain on the state being fed back. Option 43 allows the same kind of root finding for a specified damping ratio. Examples of Option 42 and 43 listings may be found in Appendix E.

#### Choosing Outputs to Feed Back

Most textbooks which address feedback control law design recommend that  $\alpha$  should be fed back to stabilize the short period mode as  $M_\alpha \doteq \omega_{sp}^2$ . This kind of stability derivative augmentation causes an increase in the short period damping ratio and undamped natural frequency. The phugoid mode is essentially not affected.

For this case, angle of attack feedback was attempted first. The X-29A ADPO advised against this, saying that the  $\alpha$  sensor was too slow for a fighter type aircraft. In cases where the  $\alpha$  sensor lags too much to be used for accurate feedback,  $N_z$  feedback is often substituted. Usually,  $\alpha$  and  $N_z$  have very similar root loci and frequency responses within the typical range of aircraft control system frequencies. In this case, they appear to be "opposites." Opposite signs

on the gain are necessary for stability.

In Fig 4 one can see that in the 180 degree root locus for  $\alpha/\delta_c$ , the locus of short period roots begins at the phugoid poles and goes to infinity, while in Fig 5 the  $N_z/\delta_c$  0 angle locus approaches its short period zeros from infinity. Because the  $N_z$  accelerometer is more accurate for feedback than the  $\alpha$  sensor, and since  $N_z$  feedback will stabilize the short period mode as well as  $\alpha$  feedback,  $N_z$  was chosen for feedback to stabilize the short period mode.

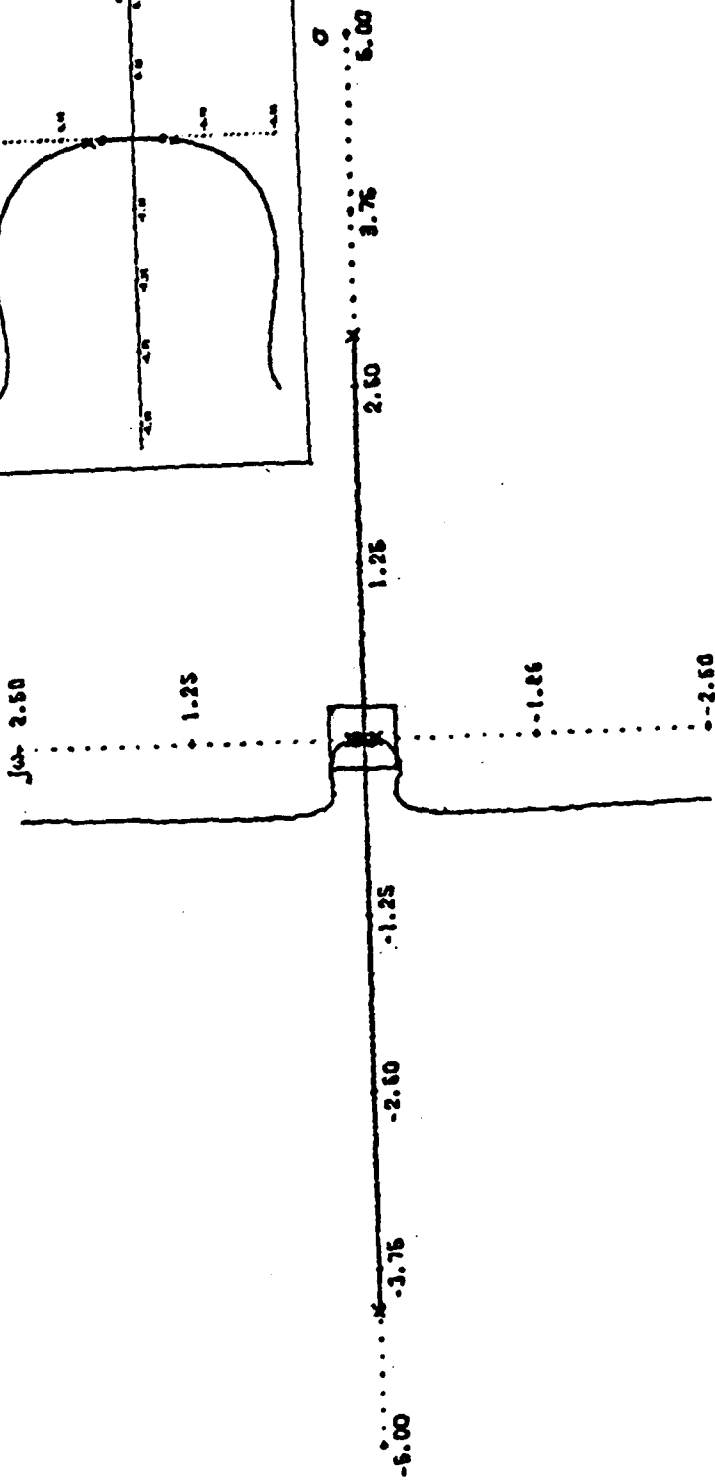
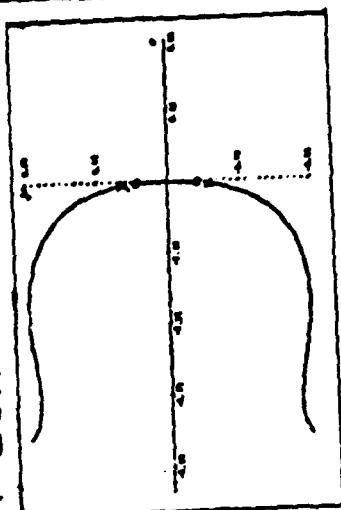
The standard method of increasing damping in the short period mode is  $q$  feedback, and that is the method applied here. Once the  $N_z$  loop has been closed and the short period roots are stable, one can see from Fig 6 that feeding back  $q$  will increase the damping ratio while decreasing the damped frequency of the short period mode.

#### Placement of the Normal Acceleration Sensor

Once it was decided that normal acceleration would be fed back to stabilize the short period mode, the accelerometer location had to be decided upon. Looking at the  $N_z/\delta_c$  transfer functions that result from locations between the nose and the tail, it was found that the zeros of the short period were most affected by sensor placement.

After looking through Flying Qualities Handbooks and data from aircraft with similar purposes, and considering past experience, a desired damped frequency of about 4 rad/sec

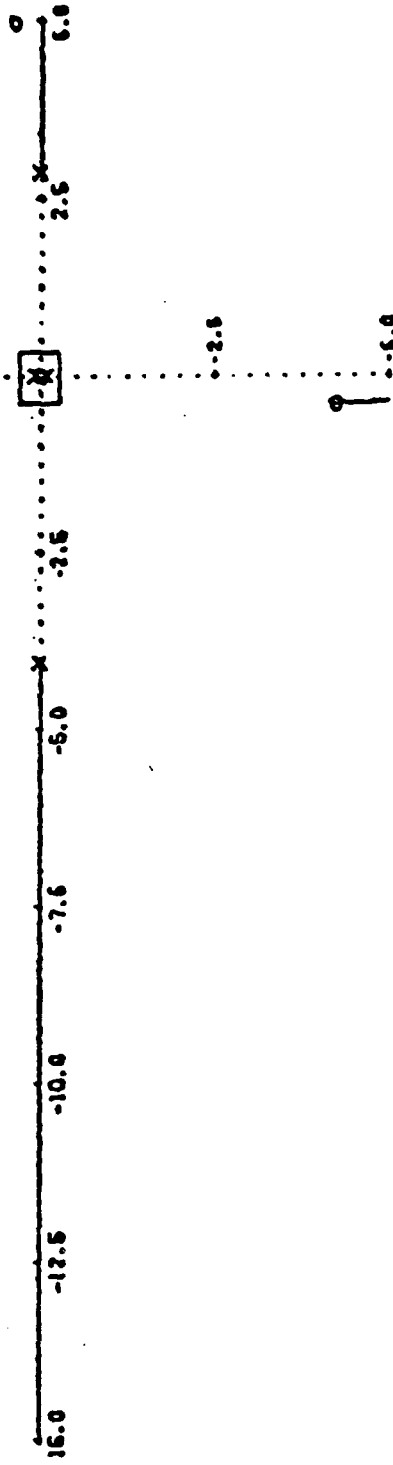
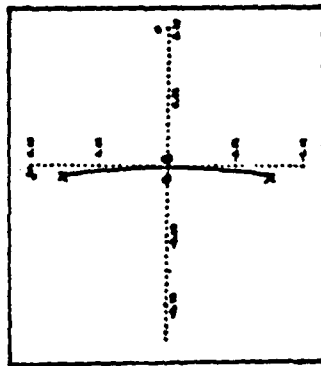
ANGLE OF ATTACK AT  $M=0.6$ .  $H=20K$  FT



$$OLTF(S) = \frac{K(S-1)(2.311S^2+0.008S+0.003)}{(S-2.867)(S+4.059)(S^2+0.014S+0.0061)}$$

Fig 4.  $\alpha/\delta_c$  at  $M=0.6$ ,  $H=20K$  ft.

NZ AT M=0.6. H=20K FT



$$Q_{LTF}(S) = \frac{-K(S+0.008)(S-0.005)(S^2-0.835S+18.082)}{(S-2.867)(S+4.059)(S^2-0.014S-0.006)}$$

Fig 5.  $N_z/\delta_c$  at M=0.6, H=20K ft



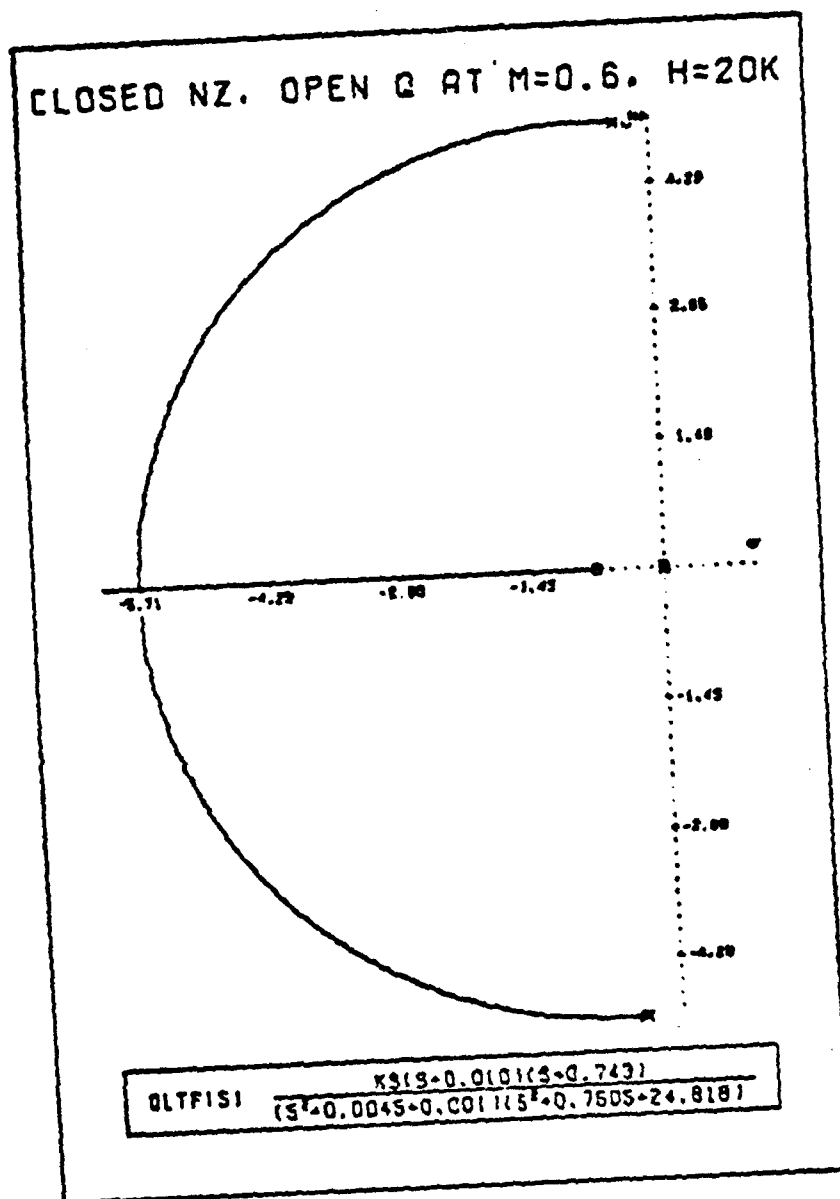


Fig 6. Closed  $N_z$ , Open  $q$  at  $M=0.6$ ,  $H=20K$  ft

for the short period was chosen. Knowing that the transfer function for the short period approximation is:

$$\frac{N_z}{\delta_c} = \frac{[Z_{\delta_c} - X_a (M_{\delta_c} + Z_{\delta_c} M_{\omega})] [s^2 + 2\zeta_{\omega_n} s + \omega_n^2]}{s^2 + 2\zeta_{sp} \omega_{n_{sp}} s + \omega_{n_{sp}}^2} \quad (4)$$

One can see that:

$$\omega_{d_{sp}} \doteq \sqrt{[M_x - (M_{\delta_c} / Z_{\delta_c}) Z_{\alpha}] / \{1 - X_a [M_{\delta_c} / Z_{\delta_c}] + M_{\omega}\}} \quad (5)$$

(10:450)

The damped frequency of the numerator zeros increase as  $X_a$ , the distance from the center of gravity to the sensor location, increases. One can see from the actual root loci of the transfer function in Appendix C, that the further forward that the normal acceleration sensor is placed with respect to the center of gravity, the higher the damped frequency of the short period open loop zeros. By looking at Figs 5 and 6 one can see that closure of the  $N_z$  loop will stabilize the short period roots and make them oscillatory, while closure of the  $q$  loop will decrease frequency and increase damping. Therefore, the location of the accelerometer was chosen so that the  $N_z / \delta_c$  zeros for the short period mode were somewhat higher than the desired short period root frequency of 4 cycles/sec. With the  $N_z$  sensor located at the 20 ft station, the short period zeros were between 2.22 and 8.90 rad/sec. The TTYLON runs for Mach = 0.6 at 20,000 ft altitude are

shown in Appendix D. With the center of gravity at 37.58 ft aft of the nose, the accelerometer was positioned 20 ft aft of the nose as it gives the short period mode a damped frequency of 4.2318 rad/sec before the  $N_z$  loop closure.

#### Gain Scheduling

For the desired damping ratio of the closed loop roots, 0.7 was chosen because it provides a fast response with a reasonable amount of overshoot. More damping will slow the system down, and less will increase the overshoot. Gain scheduling was necessary in order to achieve the desired damping ratio. Looking through the root locus for each flight condition, the gain which resulted in a 0.7 damping ratio for the closed loop short period roots was determined. These gains were plotted versus  $Q$  at each altitude and Mach number. A relationship between  $q$  and  $K_q$  can then be sought for the gain schedule.

The method used to find this relationship is the least squares fitting of data. Beginning with the basic system equation:

$$A x = b, \quad (6)$$

one may premultiply both sides by  $A^T$  and the equation is:

$$A^T A x = A^T b. \quad (7)$$

In this particular case,  $x$  is the vector  $\begin{bmatrix} C \\ D \end{bmatrix}$

where C and D are the coefficients in the equation

$$K_q = C + D Q \quad (8)$$

The vector x may be found by using

$$x = (A^T A)^{-1} A^T b \quad (9)$$

where

$$A = \begin{bmatrix} 1 & Q_1 \\ 1 & Q_2 \\ \vdots & \vdots \\ 1 & Q_M \end{bmatrix} \quad (10)$$

and

$$b = \begin{bmatrix} K_{q1} \\ K_{q2} \\ \vdots \\ K_{qM} \end{bmatrix} \quad (11)$$

This method will give the equation for the best straight line with the least amount of error squared between the data points and the chosen line.

For the second order curve fit, one is solving for the vector  $x = \begin{bmatrix} C \\ D \\ E \end{bmatrix}$  where C, D, and E are the coefficients in the equation:

$$K_q = C + DQ + EQ^2 \quad (12)$$

Eq 9 is again used to solve for x with the same b matrix as in Eq 11, but with the following A matrix:

$$A = \begin{bmatrix} 1 & Q_1 & Q_1^2 \\ 1 & Q_2 & Q_2^2 \\ \vdots & \vdots & \vdots \\ 1 & Q_M & Q_M^2 \end{bmatrix} \quad (13)$$

This equation will result in the best parabola, hyperbola, or circle to fit the data points.

#### Multiloop Closure

A method of multiloop control law design for the frequency domain is described by McGruer, Ashkenas, and Graham in Ref 10. This method allows one to achieve with equations the same results that were found using the successive loop closure method discussed in the section, "Choosing Outputs to Feed Back."

Coupling numerators are necessary for the multiloop technique when there is more than one input. For this study, there is only the canard input to the longitudinal mode response, therefore coupling numerators will not be discussed. A thorough explanation may be found in Ref 10. Fig 7 illustrates a block diagram of a flight control system's longitudinal axis with simultaneous  $N_z$  and  $q$  feedback through the canard. To derive the transfer functions for  $N_z/\delta_c$  and  $q/\delta_c$ , one must begin with the following equation:

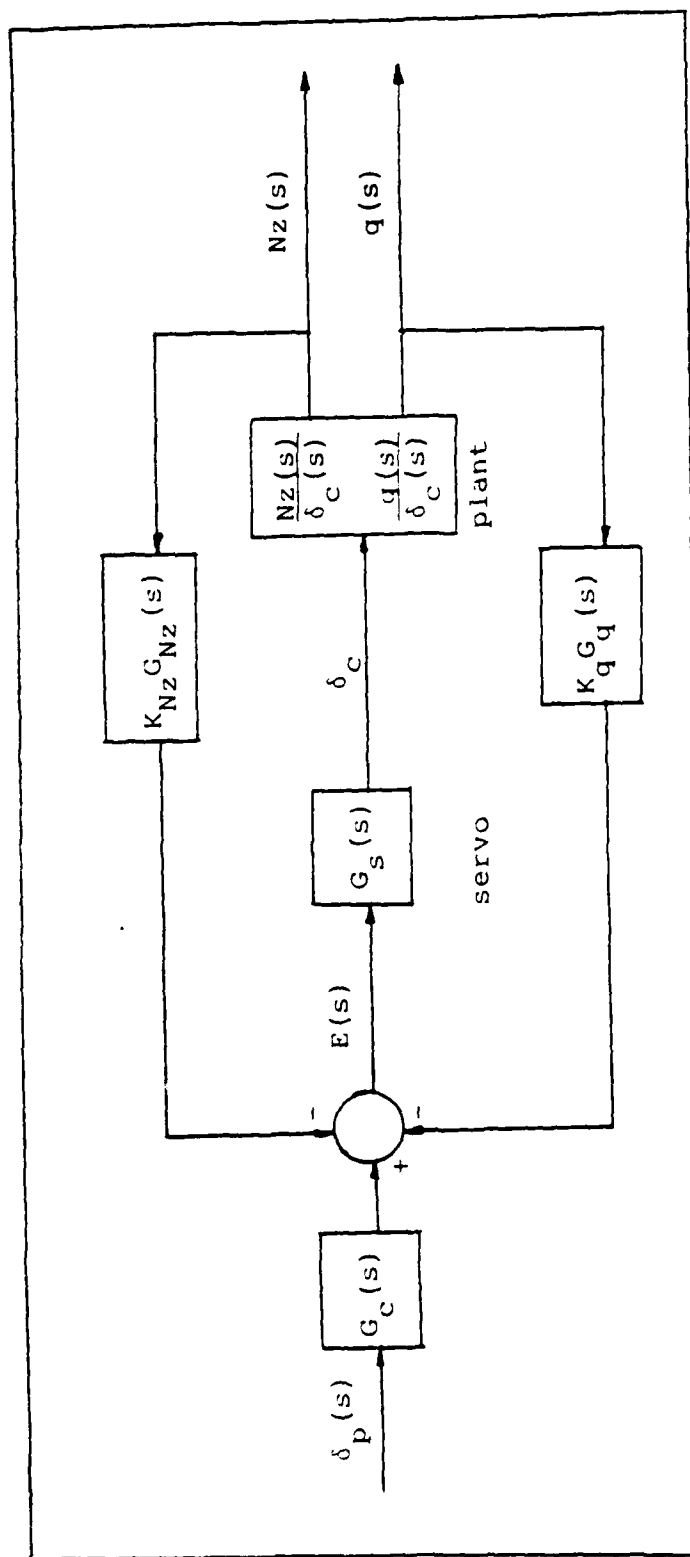


Fig 7. Block Diagram of the Longitudinal Axis of the X-29A

$$E(s) = \delta_p(s)G_c(s) - K_{N_z}G_{N_z}(s)N_z(s) - K_qG_q(s)q(s) \quad (14)$$

where:

$$N_z(s) = [N_z(s)/\delta_c(s)]_{OL}E(s)G_s(s) \quad (15)$$

and

$$q(s) = [q(s)/\delta_c(s)]_{OL}E(s)G_s(s) \quad (16)$$

Substituting Eqs 15 and 16 into Eq 14 yields:

$$\begin{aligned} E(s)\{1+K_{N_z}G_{N_z}(s)G_s(s)[N_z(s)/\delta_c(s)]_{OL} \\ + K_qG_q(s)G_s(s)[q(s)/\delta_c(s)]_{OL}\} = \delta_p(s)G_c(s) \end{aligned} \quad (17)$$

The command and servo transfer functions,  $G_c(s)$  and  $G_s(s)$ , have been assumed to be 1 as was explained in the section "Deriving the Transfer Functions." This means that  $E(s) = \delta_p(s) = \delta_c(s)$ . The two multiloop transfer functions then simplify to:

$$[N_z(s)/\delta_c(s)]_{CL} = \frac{[N_z(s)/\delta_c(s)]_{OL}}{1+K_{N_z}[N_z(s)/\delta_c(s)]_{OL}+K_q[q(s)/\delta_c(s)]_{OL}} \quad (18)$$

and

$$[q(s)/\delta_c(s)]_{CL} = \frac{[q(s)/\delta_c(s)]_{OL}}{1+K_{N_z}[N_z(s)/\delta_c(s)]_{OL}+K_q[q(s)/\delta_c(s)]_{OL}} \quad (19)$$

### III. Design

#### Multiloop Design

The gain chosen for  $N_z$  was fed back and the  $N_z$  closed loop roots were found using Option 42 of TOTAL. From Fig 7, one can see that when the  $N_z$  loop is closed, this transfer function is:

$$\left[ \frac{N_z}{\delta_c} \right]_{CL} = \frac{N_1 D_2}{N_1 N_2 + D_1 D_2} = \frac{N_{znum}}{K_{N_z} N_{znum} + \Delta_{OL}} \quad (20)$$

$$= \frac{(As^4 + Bs^3 + Cs^2 + Ds + E)}{K_{N_z} (As^4 + Bs^3 + Cs^2 + Ds + E) + (s^4 + as^3 + bs^2 + cs + d)} \quad (21)$$

The highest order coefficient in the denominator becomes  $(K_{N_z} A + 1)$ . The effective  $q/\delta_c$  open loop transfer function is then:

$$\left[ \frac{q}{\delta_c} \right]_{OL} = \frac{q_{num}}{(K_{N_z} A + 1) (N_z \text{ closed loop roots})} \quad (22)$$

A gain  $K_q$  is then found which meets the desired damping ratio requirements. After finding the  $q$  closed loop roots with Option 42, the closed loop transfer function is found:



$$\left[ \frac{q}{\delta_c} \right]_{CL} = \frac{N_1 D_2}{N_1 N_2 + D_1 D_2} = \frac{q_{num}}{K_q q_{num} + (K_{N_z} A + 1) (N_z \text{ closed loop roots})} \quad (23)$$

$$= \frac{q_{num}}{K_q (Ws^3 + Xs^2 + Ys + Z) + (K_{N_z} A + 1) (s^4 + ps^3 + qs^2 + rs + t)} \quad (24)$$

$$= \frac{q_{num}}{(K_{N_z} A + 1) (q \text{ closed loop roots})} \quad (25)$$

This is the overall closed loop  $q/\delta_c$  transfer function. The overall  $N_z/\delta_c$  transfer function of course has the same characteristic equation and the original  $N_z$  numerator:

$$\left[ \frac{N_z}{\delta_c} \right]_{CL} = \frac{N_{z_{num}}}{(K_{N_z} A + 1) (q \text{ closed loop roots})} \quad (26)$$

The  $N_z$  loop was closed first because this feedback stabilized the short period mode. Any kind of special flight control modes such as altitude hold, or speed control which may be desired can be wrapped around the  $N_z$  closed loop now that the system is stable. In this case the pitch rate feedback was wrapped around the  $N_z$  feedback to make the system meet Level I Flying Qualities.

#### Selection of Gains

An attempt was made to find a  $K_{N_z}$  gain schedule which would result in a constant damped frequency of the short period roots. The highest frequency for which this is

possible with a stable short period mode is 4.7 rad/sec. Mach = 0.5,  $h = 40,000$  ft is the limiting flight condition at which the short period is unstable at damped frequencies higher than 4.7 rad/sec. Fig 8 illustrates this. At all the higher Mach flight conditions, the open loop  $N_z/\delta_c$  root locus is limited by its short period zero locations and will not go to a damped frequency as low as 4.7 rad/sec. At Mach = 0.8, sealevel, the lowest frequency attainable is 8.9 rad/sec. Fig 9 illustrates this problem. As was discussed in the section, "Placement of Normal Acceleration Sensor," the  $N_z$  accelerometer location is the factor which drives this frequency flexibility.

Since constant damped frequency was not possible, another simple approach to  $N_z$  feedback was attempted. It was decided that a constant gain,  $K_{N_z}$ , would be found to stabilize the short period mode for all flight conditions and that the short period damped frequency would be allowed to vary.  $K_{N_z} = -1.2$  was chosen as it was the lowest gain which would stabilize the limiting condition Mach = 0.5,  $h = 40,000$  ft. This can be seen in the root loci in Appendix E. With the  $N_z$  loop closed and the accelerometer at the 20 ft station, the damped frequencies varied from 4.01 rad/sec to 9.21 rad/sec. These results of the  $N_z$  loop closure are shown in Table III.

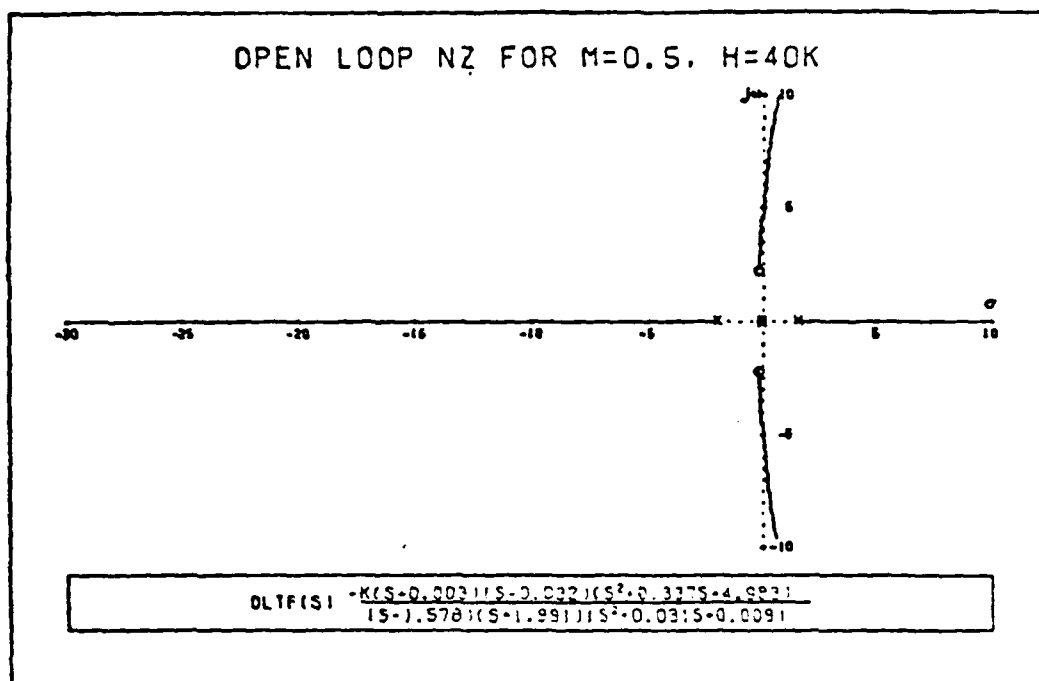


Fig 8.  $N_z/\delta_c$  at M=0.5, H=40K ft

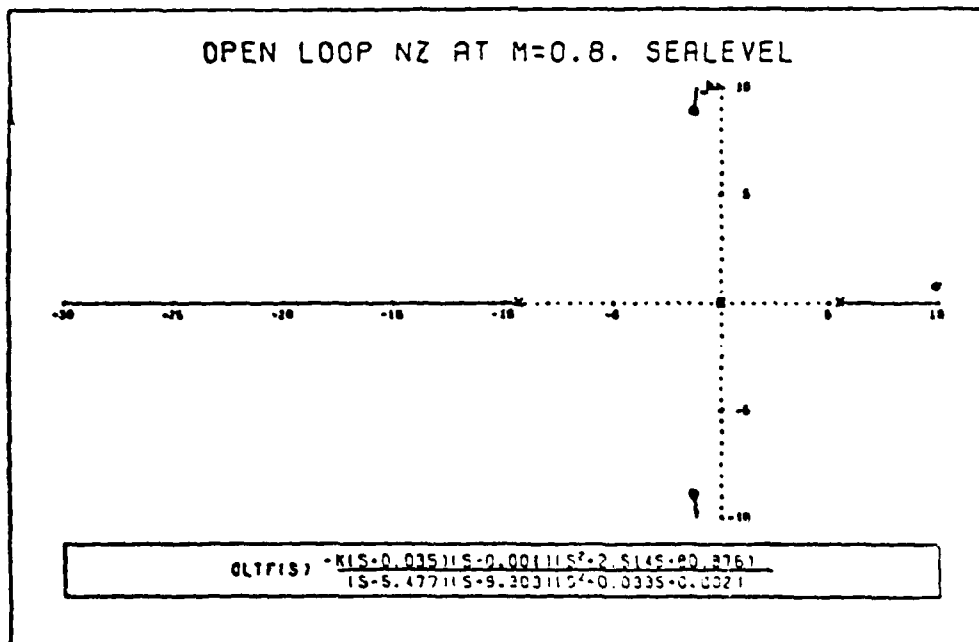


Fig 9.  $N_z/\delta_c$  at M=0.8, Sealevel

Table III  
Roots Resulting From  $K_{N_z} = -1.2$

Flight Condition		Short Period		Phugoid	
M	h(ft)	real	imag	real	imag
0.3	0	-0.29209	4.0147	0.43183e-2	0.60757e-1
0.6	0	-0.82973	6.7733	-0.98176e-2	0.91793e-2
0.8	0	-1.2305	9.2053	-0.33280e-1 -0.64486e-3	0 0
0.4	20K	-0.17979	4.0600	0.46977e-2	-0.52395e-1
0.6	20K	-0.37512	4.9676	-0.19634e-2	-0.24173e-1
1.0	20K	-0.56668	6.5666	-0.24510e-1 0.18225e-2	0 0
1.2	20K	-0.56932	6.6404	-0.34739e-1 0.32160e-2	0 0
0.5	40K	-0.11778e-1	4.7494	-0.54698e-2	-0.51752e-1
0.6	40K	-0.11849	4.2240	0.40106e-2	0.38340e-1
1.0	40K	-0.23649	4.7065	-0.37698e-2	0.12623e-1
1.4	40K	-.26632	4.9925	-0.15595e-1 -0.36254e-3	0 0

One can see that the short period mode has now been stabilized. The damping ratios range from 0.002 to 0.13. This is much too low, and an increase in damping must be achieved.

As was discussed in the section, "Gain Scheduling,"  $K_q$  was chosen to try to achieve  $\delta = 0.7$  for all short period roots. Table IV shows the  $Q$  vs  $K_q$  schedule that provided the 0.7 damping ratio desired.

Table IV

 $K_q$  to Achieve  $\gamma = 0.7$  at Selected Flight Conditions

M	Flight Condition		$K_q$
	h(ft)	Q(lb/ft <sup>2</sup> )	
0.5	40K	69	2.08
0.6	40K	99	3.00
0.4	20K	109	3.18
0.3	0	134	3.65
0.6	20K	246	5.25
1.0	40K	275	5.98
0.6	0	535	8.55
1.4	40K	539	10.25
1.0	20K	682	10.08
0.8	0	951	12.21
1.2	20K	982	13.45

Looking at the data points plotted in Fig 10, one can see that there are a few curves which look like they may fit the points. In this study, three methods were tried. First, a linear curve fit using all the data points was used. Second, a quadratic curve fit was tried, and finally, the points were broken into two groups. One line was fit through the high Q group and another through the low Q group. These methods shall be referred to as Method I, II, and III, respectively. Using the method of least squares described in the

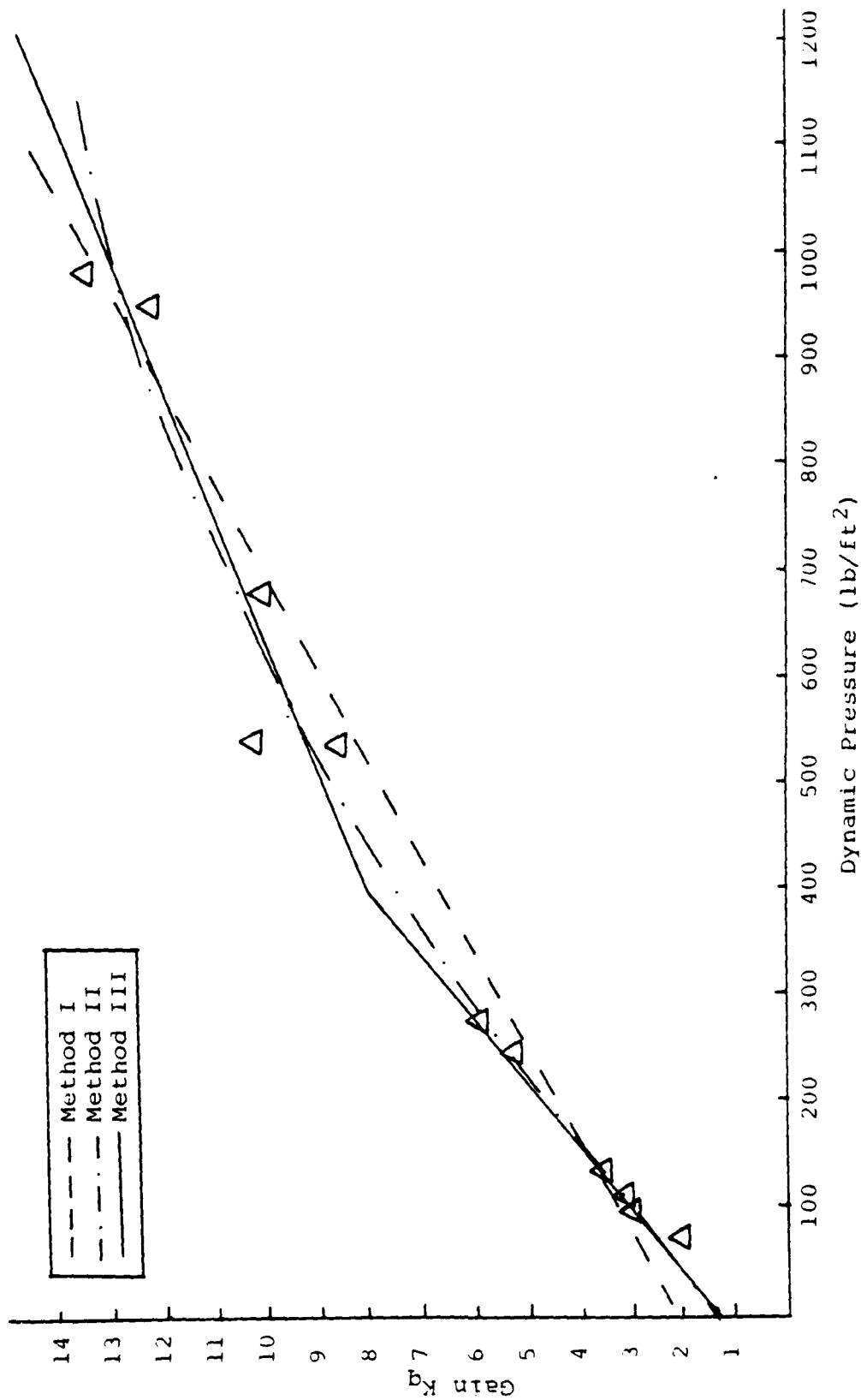


Fig 10.  $K_q$  vs Dynamic Pressure Curves for the Three Curve Fitting Methods

section, "Gain Scheduling," the vector of coefficients

$x = \begin{bmatrix} C \\ D \end{bmatrix}$  or  $x = \begin{bmatrix} C \\ D \\ E \end{bmatrix}$  was solved for. The work can be found in

Appendix F. In Method I, the equation for the line which results is:

$$K_q = 2.081 + 0.01142Q \quad (27)$$

Using a second order curve fit in Method II, the equation for  $K_q$  is:

$$K_q = 1.14 + 0.01866Q - 0.000006849Q^2 \quad (28)$$

Grouping the data points in Fig 10 into the six lower and the five higher points, the two lines which result from Method III are:

For  $Q$  less than or equal to  $405 \text{ lb/ft}^2$

$$K_q = 1.174 + 0.01717Q \quad (29)$$

For  $Q$  greater than  $405 \text{ lb/ft}^2$

$$K_q = 4.839 + 0.008226Q \quad (30)$$

The damping ratios for the closed loop roots using the gains found by each of these methods are shown in Table V.

Table V  
Damping Ratios for Closed Loop Roots Resulting  
From the Three Curve Fitting Methods

Flight Condition			$\zeta$		
M	h	Q(lb/ft <sup>2</sup> )	Method I	Method II	Method III
0.3	0	134	0.69	0.72	0.67
0.6	0	535	0.68	0.75	0.74
0.8	0	951	0.73	0.73	0.72
0.4	20K	109	0.73	0.73	0.68
0.6	20K	246	0.66	0.74	0.72
1.0	20K	682	0.69	0.73	0.72
1.2	20K	982	0.69	0.69	0.68
0.5	40K	69	0.95	0.88	0.79
0.6	40K	99	0.74	0.74	0.67
1.0	40K	275	0.62	0.70	0.69
1.4	40K	539	0.58	0.65	0.64

The desired damping ratio is 0.7. Below are listed the mean and standard deviation for each set of points.

Table VI  
Determination of Goodness of Fit For  
Three Curve Fitting Methods

	Method I	Method II	Method III
Mean	0.7054	0.7327	0.7018
Standard Deviation	0.0897842	0.0539509	0.0395006



One can see that Method III has the mean which is closest to 0.7 and has the least standard deviation from it. Therefore, the best linear fit to the  $K_q$  curve is:

If  $Q$  is less than or equal to  $405 \text{ lb/ft}^2$

$$K_q = 1.174 + 0.01727Q \quad (29)$$

And if  $Q$  is greater than  $405 \text{ lb/ft}^2$

$$K_q = 4.839 + 0.008226Q \quad (30)$$

These equations can be put into the analog computer to control the gain changes. The roots that result from this schedule can be seen in Table VII.

Table VII  
Closed Loop Roots at Design Flight Conditions

Flight Condition		Phugoid		Short Period		
M	h	real	imag	real	imag	$\zeta$
0.3	0	0.26956e-2	0.54711e-1	3.0201	3.3035	0.67
0.6	0	-0.10159e-1	0.55097e-2	-5.8805	5.3295	0.74
0.8	0	-0.30701e-1 -0.51049e-3	0 0	-7.8115	7.5555	0.72
0.4	20K	-0.37562e-2	0.48607e-1	-2.9694	3.2268	0.68
0.6	20K	-0.24807e-2	0.21861e-1	-3.9429	3.8222	0.72
1.0	20K	-0.24323e-1 0.14568e-2	0 0	-5.3280	5.1358	0.72
1.2	20K	-0.33995e-1 0.25804e-2	0 0	-5.1033	5.5252	0.68
0.5	40K	0.50595e-2	0.49254e-1	-3.9592	3.0402	0.79
0.6	40K	0.35963e-2	0.36407e-1	-3.0001	3.2901	0.67
1.0	40K	-0.39368e-2	0.11629e-1	-3.5106	3.6391	0.69
1.4	40K	-0.30269e-3	0	-3.4927	4.1618	0.64

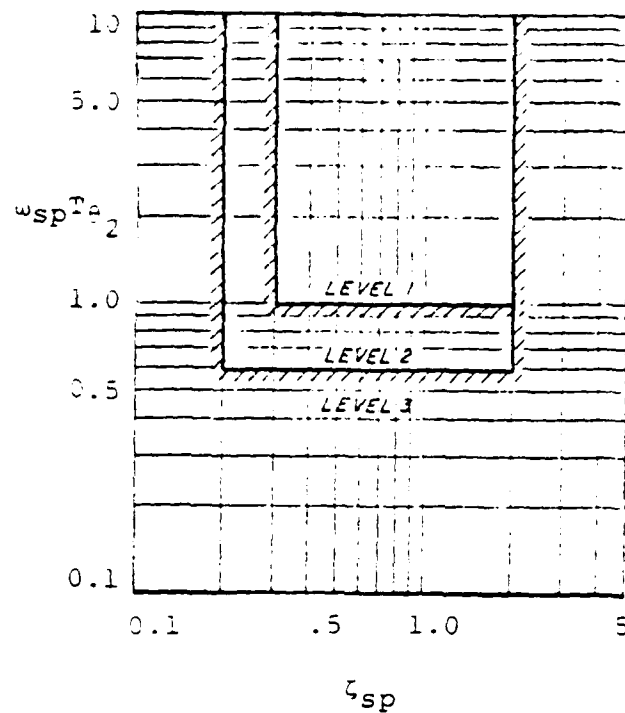


Fig 11. Short Period Mode Flying Qualities Requirement

The choice of  $\omega_{sp} = 4$  and  $\zeta_{sp} = 0.7$  was done mainly from past experience and a knowledge of the handling qualities specifications. Now the results will be tested for Level I Flying Qualities. Using Fig 11, one can see that the requirements are that  $\zeta_{sp}$  lies between 0.25 and 1.1 and  $\omega_{sp} T_{\theta_2}$  lies above 1.5. Table VIII shows the values of  $\omega_{sp} T_{\theta_2}$  and  $\zeta_{sp}$  for this design.

Table VIII  
Short-Term Pitch Response to Pitch Controller  
( $\omega_{sp}T_{\theta_2}$  vs.  $\zeta_{sp}$ )

Flight Condition		$\omega_{sp}$	$T_{\theta_2}$	$\omega_{sp}T_{\theta_2}$	$\zeta$
Mach	h(ft)				
0.3	0	3.3035	9.1889	30.3555	0.67
0.6	0	5.3295	3.8709	20.6298	0.74
0.8	0	7.5555	2.5883	19.5562	0.72
0.4	20K	3.2268	13.378	43.1686	0.68
0.6	20K	3.8222	8.4619	32.3429	0.72
1.0	20K	5.1358	5.2871	27.1535	0.72
1.2	20K	5.5252	4.6868	25.8958	0.68
0.5	40K	3.0402	22.230	67.5823	0.79
0.6	40K	3.2901	19.032	62.6187	0.67
1.0	40K	3.6391	12.306	44.7836	0.69
1.4	40K	4.1618	8.9739	37.3477	0.64

When these values are plotted as in Fig 12, one can see that the short period performance meets Level I standards.

#### Phugoid Design Modification

One can see from Table VII that half of the flight conditions with the  $N_z$  loop closed have an unstable phugoid mode. This occurred when  $K_{N_z}$  was fed back to stabilize the short period mode because the  $N_z/s_C$  transfer function has

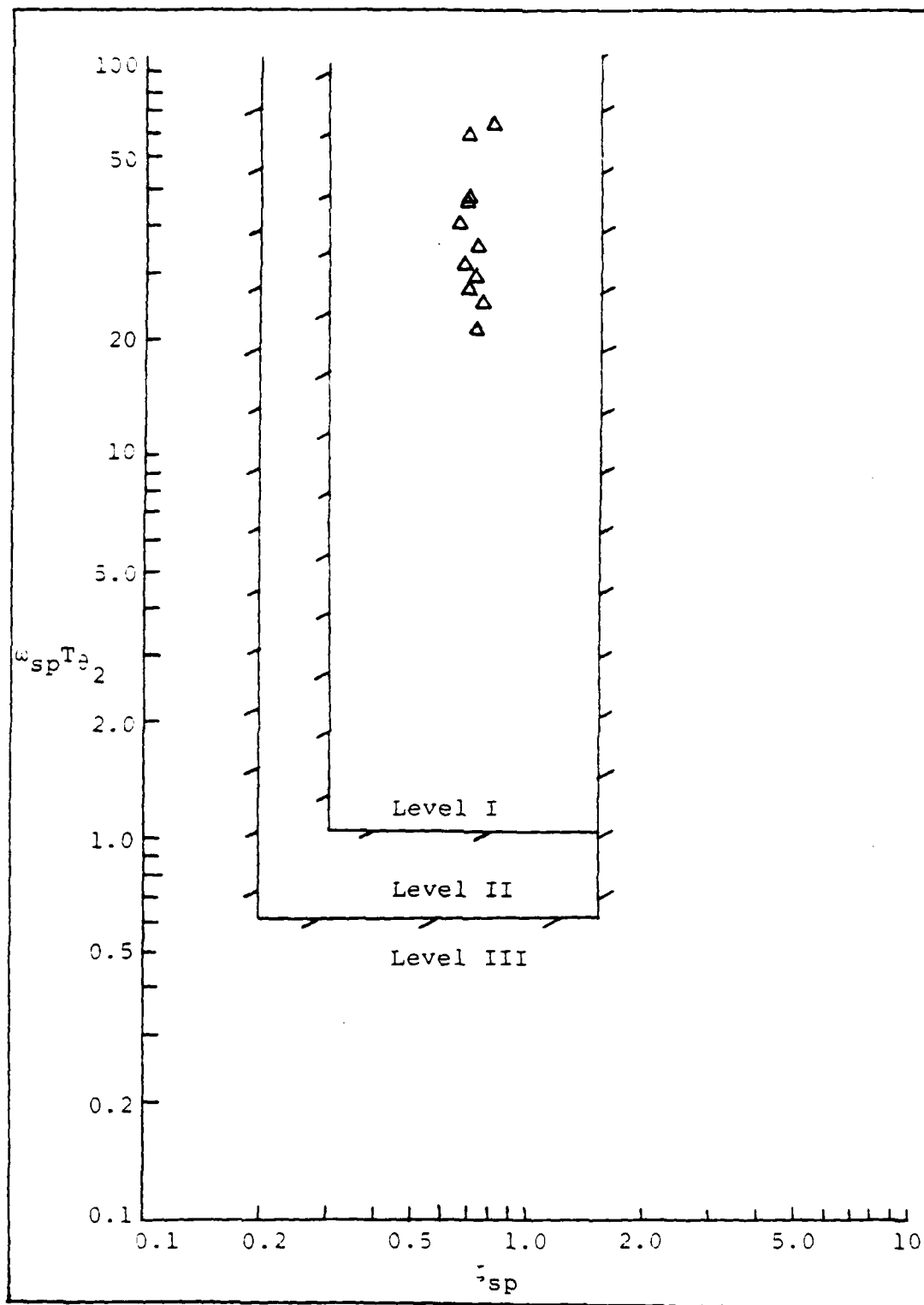


Fig 12.  $\omega_{sp} T_{e2}$  vs  $\zeta_{sp}$  for Design Results

zeros in the right half plane. At that time it was decided that the stability of the short period mode was of greater importance than the phugoid mode because of the faster response of the former. The  $N_2$  feedback which resulted in the phugoid instability was used. The Flying Qualities Specification requires the time to double amplitude for the phugoid mode be greater than 55 sec for Level III. Time response plots such as the one in Fig 13 show that the phugoid mode time to double amplitude for this design is between 250 and 300 sec. Since this is a fighter aircraft and the pilot would be expected to give the aircraft a new input at least every minute, one can say that the time to double amplitude restriction is sufficient. However, to have Level I flying qualities, the phugoid mode must be stable and have a damping ratio of at least 0.04. To meet this standard, some kind of filter must be added.

Using the Mach = 0.3, sealevel flight condition for an example, a filter has been designed to show how the phugoid mode can be stabilized. The gain,  $K_q$ , which, according to the gain schedule, will be fed back at this flight condition is -3.483. Looking at the listing of the root locus of the uncompensated system with the  $N_2$  loop closed and the  $q$  loop open, one can see that the phugoid mode is presently unstable at gains greater than -13.0657. To stabilize it, a lag network has been introduced in cascade with the plant. The pole

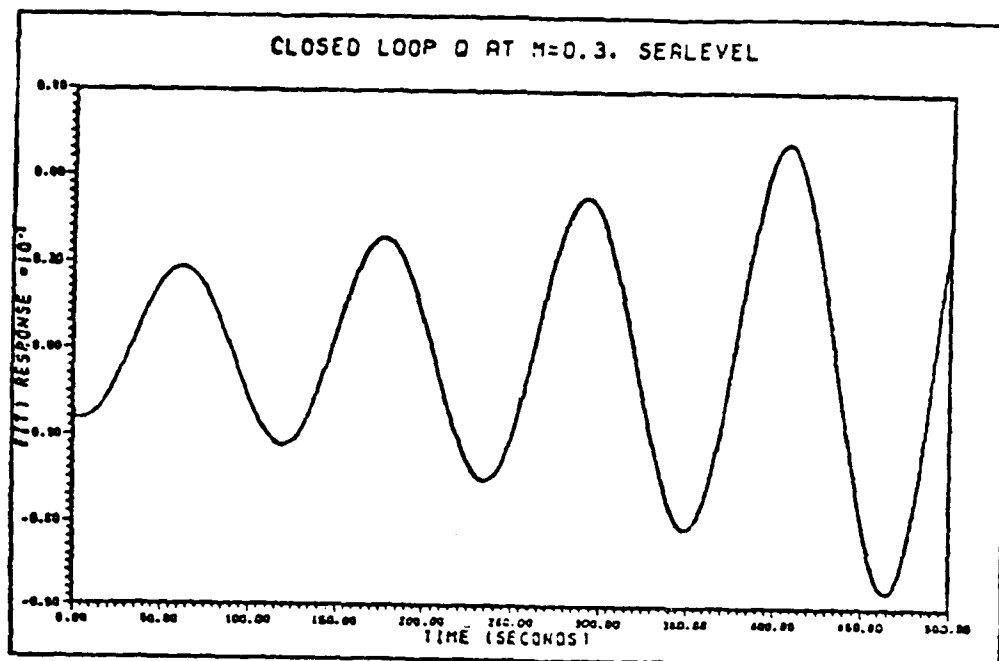


Fig 13. Time Response for  $q/\delta_c$  at  $M=0.3$ , Sealevel

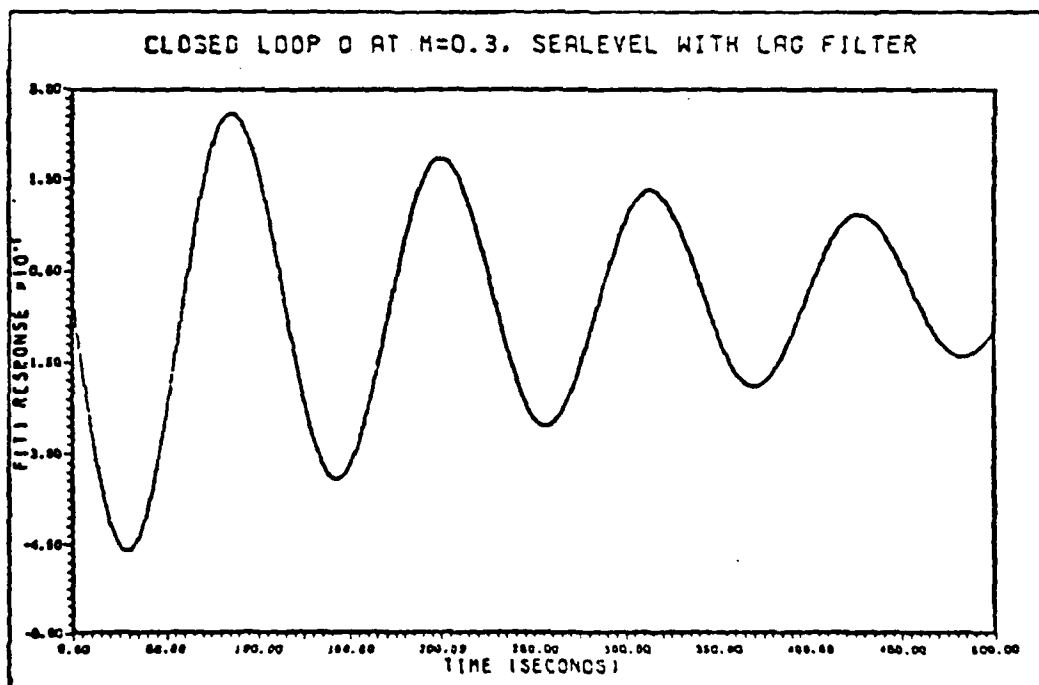


Fig 14. Time Response for  $q/\delta_c$  With Lag Compensation at  $M=0.3$ , Sealevel

of the filter has been placed close to the origin at  $s = -0.002 + j0$ . The zero of the filter was placed to the left of the zero at  $-0.12823 + j0$  in order to bring the locus left. After a few trials, the lag filter chosen is then:

$$G_C(s) = \frac{s + 1/T}{s + 1/\alpha T} \quad (31)$$

where

$$T = 16.67 \text{ sec}$$

$$\alpha = 30$$

so

$$G_C(s) = \frac{s + 0.06}{s + 0.002} \quad (32)$$

Now one can see from Table IX that the phugoid mode is stable below a gain,  $K_q = -1.9427$ . This means that for the gain  $K_q = -3.483$  which will be fed back to meet short period mode requirement for  $\zeta = 0.7$ , the phugoid mode will also be stable.



Table IX  
Root Locus for Phugoid Mode With and  
Without Lag Filter

Without Lag Filter			With Lag Filter		
Real	Imag	Gain	Real	Imag	Gain
0.43183e-2	0.60757e-1	0	0.43183e-2	0.60757e-1	0
0.17287e-2	0.51098e-1	-6.2058	0	0.57632e-1	-1.9417
0	0.44441e-1	-13.066	-0.14470e-1	0.43869e-1	-14.744
-0.22891e-2	0.34706e-1	-30.938	-0.25280e-1	0.27091e-1	-47.912
-0.41647e-2	0.24884e-1	-72.668	-0.31051e-1	0.80022e-2	-122.40
-0.54938e-2	0.14973e-1	-205.25	-0.31600e-1	0	-138.89
-0.61827e-2	0.49968e-2	-873.63	-0.51600e-1	0	-329.21
0	0	0	-0.60000e-1	0	0

Option 42 in TOTAL was used again to determine the roots of  $q/\delta_c$  with the lag filter at the desired  $K_q = -3.483$  for this flight condition. These roots are:

Filter	$s = -0.0019251$
Short Period	$s = -3.0092 \pm j3.3502$
Phugoid	$s = -0.0027732 \pm j0.055429$

This phugoid mode has a damping ratio of 0.04997, which means that it has Level I flying qualities. The time response of the longitudinal mode with the lag filter is shown in Fig 14. The root loci that show the phugoid mode with

and without the filter may be seen in Figs 15 and 16. A design for only one flight condition is shown here. If it is decided that the phugoid mode must be stable and meet Level I flying qualities at all points in the flight envelope, then this procedure must be followed for all flight conditions.

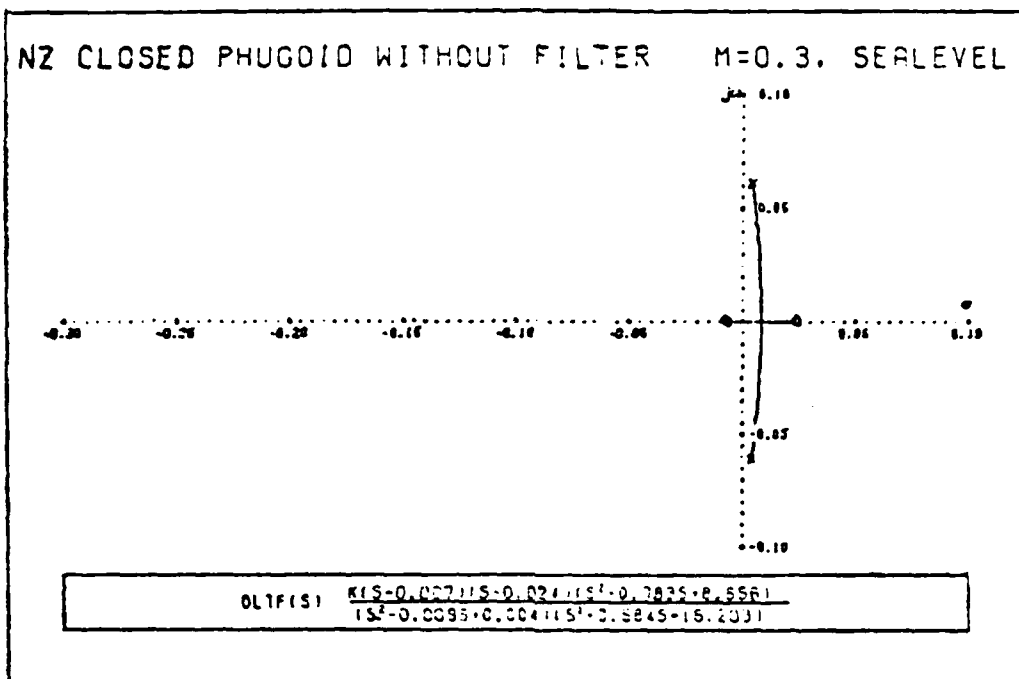


Fig 15. Closed  $N_2$ , Open  $q$  Loop Phugoid Without Filter at  $M=0.3$ , Sealevel

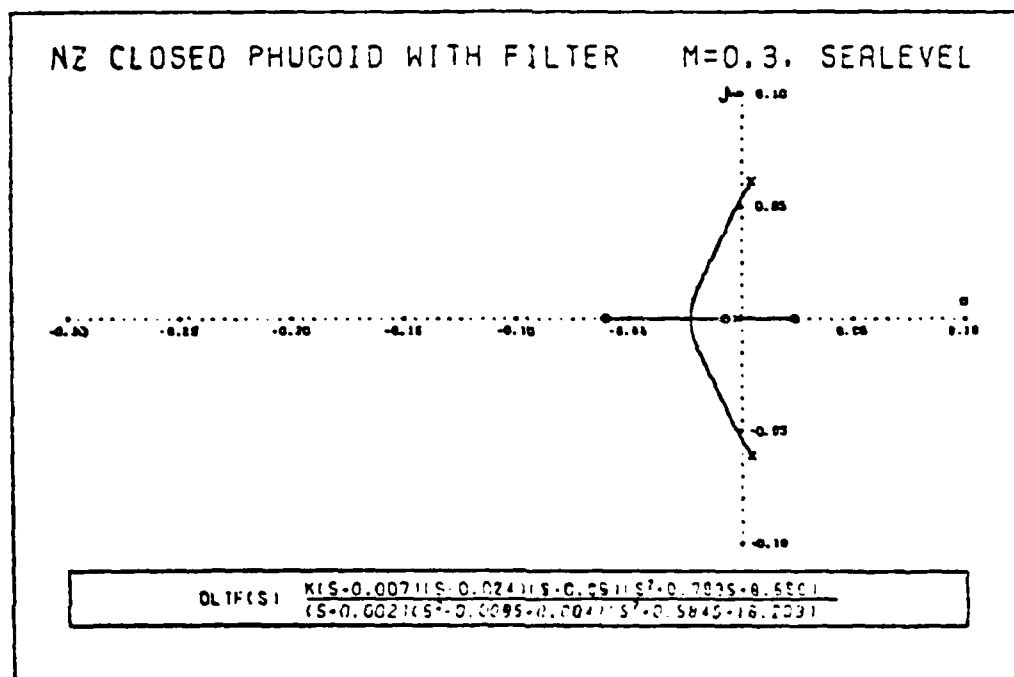


Fig 16. Closed  $N_2$ , Open  $q$  Loop Phugoid With Filter at  $M=0.3$ , Sealevel

#### IV. Conclusions and Recommendations

This study has shown that an analog backup flight control system which results in Level I handling qualities can be designed. Attention must be paid to the placement of the normal acceleration sensor to get good short period performance. To stabilize and improve the short period mode, simple gain feedback of the  $N_z$  and  $q$  states were used. Normal acceleration was fed back first to stabilize the short period mode and then pitch rate feedback was added to improve performance to Level I Flying Qualities standards. To stabilize the phugoid mode a lag filter may be added.

This design was done using a rigid body assumption. Further study could be done by looking at the aeroelastic effects of the longitudinal axis. Other design methods may also be used to compare complexity and performance. A design of the control laws for the lateral-directional axis should be done with aeroelastic effects considered.

### Bibliography

1. D'Azzo, John J. and Constantine H. Houppis. Linear Control System Analysis and Design. New York: McGraw-Hill Book Company, 1981.
2. Grumman Aerospace Corporation. Forward Swept Wing Demonstrator Technology Integration and Evaluation Study. AFWAL-TR-80-3145. Vol I; Wright-Patterson AFB, Ohio: Flight Dynamics Laboratory, June 1980.
3. Grumman Aerospace Corporation. Forward Swept Wing Demonstrator Technology Integration and Evaluation Study. AFWAL-TR-80-3145. Vol II; Wright-Patterson AFB, Ohio: Flight Dynamics Laboratory, June 1980.
4. Heffley, Bob and Wayne Jewwell. Aircraft Handling Qualities Data. NASA-CR-2144. Hawthorne, California: Systems Technology Incorporated, December 1972.
5. Hogg, Robert V. and Allen T. Craig. Introduction to Mathematical Statistics. New York: Macmillan Publishing Co., Inc., 1978.
6. Hoh, et al. Proposed Mil Standard and Handbook - Flying Qualities of Air Vehicles - Vol II: Proposed MIL Handbook. AFWAL-TR-82-3081. Wright-Patterson AFB, Ohio: Flight Dynamics Laboratory, July 82.
7. House, Eric B. II. Investigation of the YF-16 in High Angle of Attack Asymmetric Flight. MS Thesis. AFIT/GAE/AA/78M-6. Wright-Patterson AFB, Ohio: School of Engineering, March 1978.
8. Larimer, Stanley J. TOTAL - An Interactive Linear Control Design and Analysis Program. AFIT/ENE, Wright-Patterson AFB, OH.
9. McGlynn, Henry J. TTYLON - A Longitudinal Aircraft Transfer Function Program. ASD/ENFTC, Wright-Patterson AFB, OH.
10. McRuer, Duane, Irving Ashkenas, and Dunstan Graham. Aircraft Dynamics and Automatic Control. Princeton: Princeton University Press, 1973.
11. Roskam, Jan. Airplane Flight Dynamics and Automatic Flight Controls - Part I. Lawrence, Kansas: Roskam Aviation and Engineering Corp., 1979.

12. Roskam, Jan. Airplane Flight Dynamics and Automatic Flight Controls - Part II. Lawrence, Kansas: Roskam Aviation and Engineering Corp., 1979.
13. Strang, Gilbert. Linear Algebra and Its Applications. New York: Academic Press, 1980.
14. MIL-F-8785C. Military Specification: Flying Qualities of Piloted Airplanes. November 1980.

## Appendix A

### Wind Tunnel Data

Wind tunnel data in the form of weights, geometry, moments of inertia, stability derivatives, and drag data were used in the TTYLON program to get the X-29A longitudinal transfer functions. This data follows.









1  
2

Latitude

Mean  
1.2 -7323.1  
1.4 -7156.1  
1.6 -7176.1  
1.8 -7467.1  
2.0 -9067.1  
2.2 -9037.1  
2.4 -7204.1  
2.6 -6011.1  
2.8 -6102.1  
3.0 -6141.1

1  
2

Latitude 0

100

200

300

400

500

Mean  
1.2 1.00 0.00  
1.4 1.00 0.00  
1.6 1.00 0.00  
1.8 1.00 0.00  
2.0 1.00 0.00  
2.2 1.00 0.00  
2.4 1.00 0.00  
2.6 1.00 0.00  
2.8 1.00 0.00  
3.0 1.00 0.00

1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00

1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00

1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00

1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00

1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00  
1.00 0.00

For  $\Delta t_{\text{pulse}} = 10^{-6}$  s

Mach = 0.2

$\theta$	$\tau$
0.0	0.024
0.1	0.041
0.2	0.124

Mach = 0.6

$\theta$	$\tau$
0.1	0.033
0.2	0.117
0.3	0.173

Mach = 0.8

$\theta$	$\tau$
0.0	0.033
0.1	0.052
0.2	0.140

Mach = 1.0

$\theta$	$\tau$
0.1	0.053
0.2	0.079
0.3	0.231

Appendix B  
TTYLON Flow Diagrams

The computer program TTYLON, which is maintained by ASD/ENFTC at Wright-Patterson AFB, was used to transform wind tunnel data into transfer functions to model the X-29A. Several flow diagrams follow which show the way the program works. First, an overall, top level diagram is shown. Next, the weight and moment of inertia input procedure is portrayed. The last two flow diagrams show the method by which the aerodynamic data is input and the way in which a transfer function for each desired flight condition is produced. The dotted lines indicate the options which were used in this design.

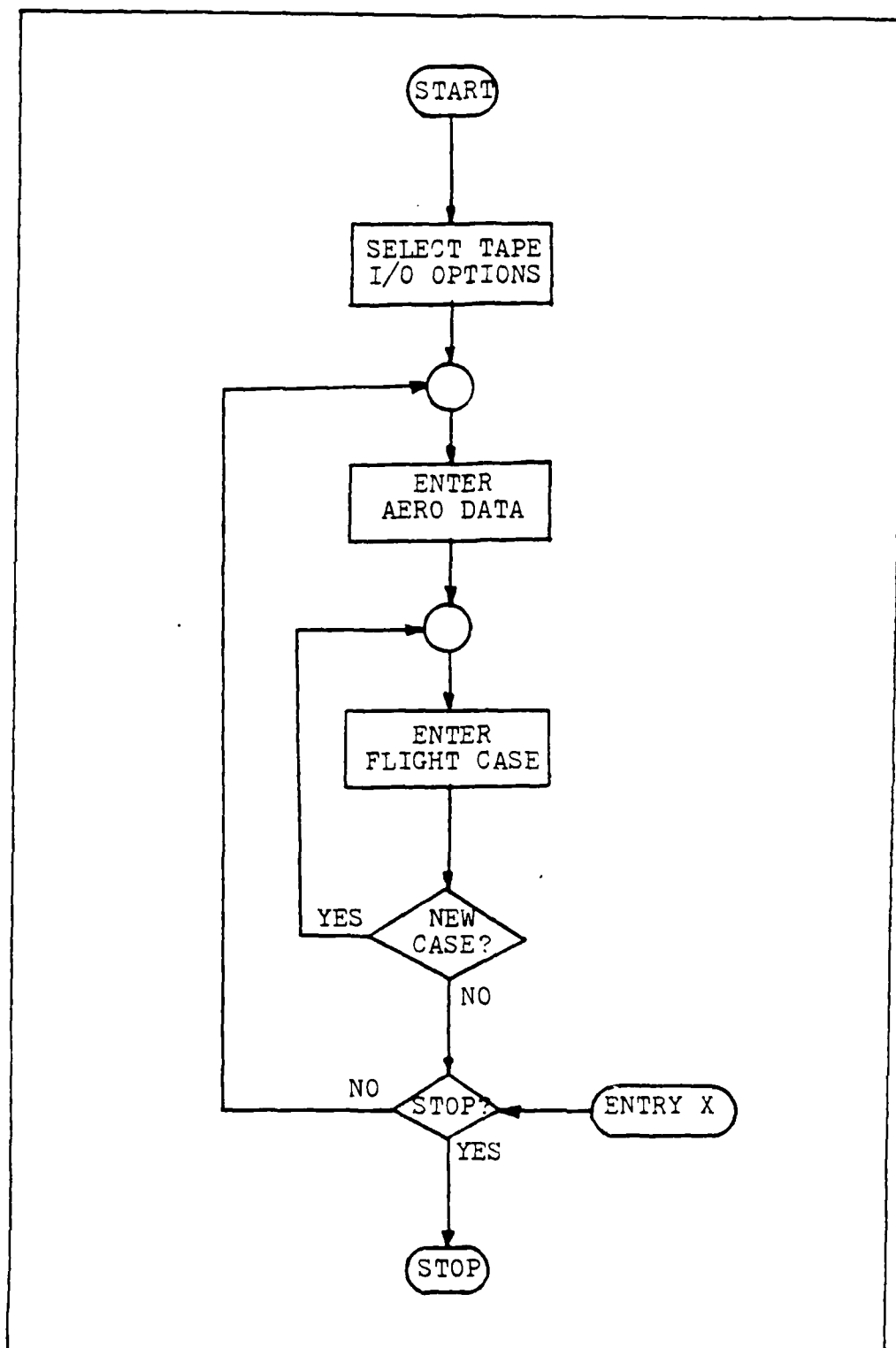


Fig B-1. Overall TTYLON Flow Diagram

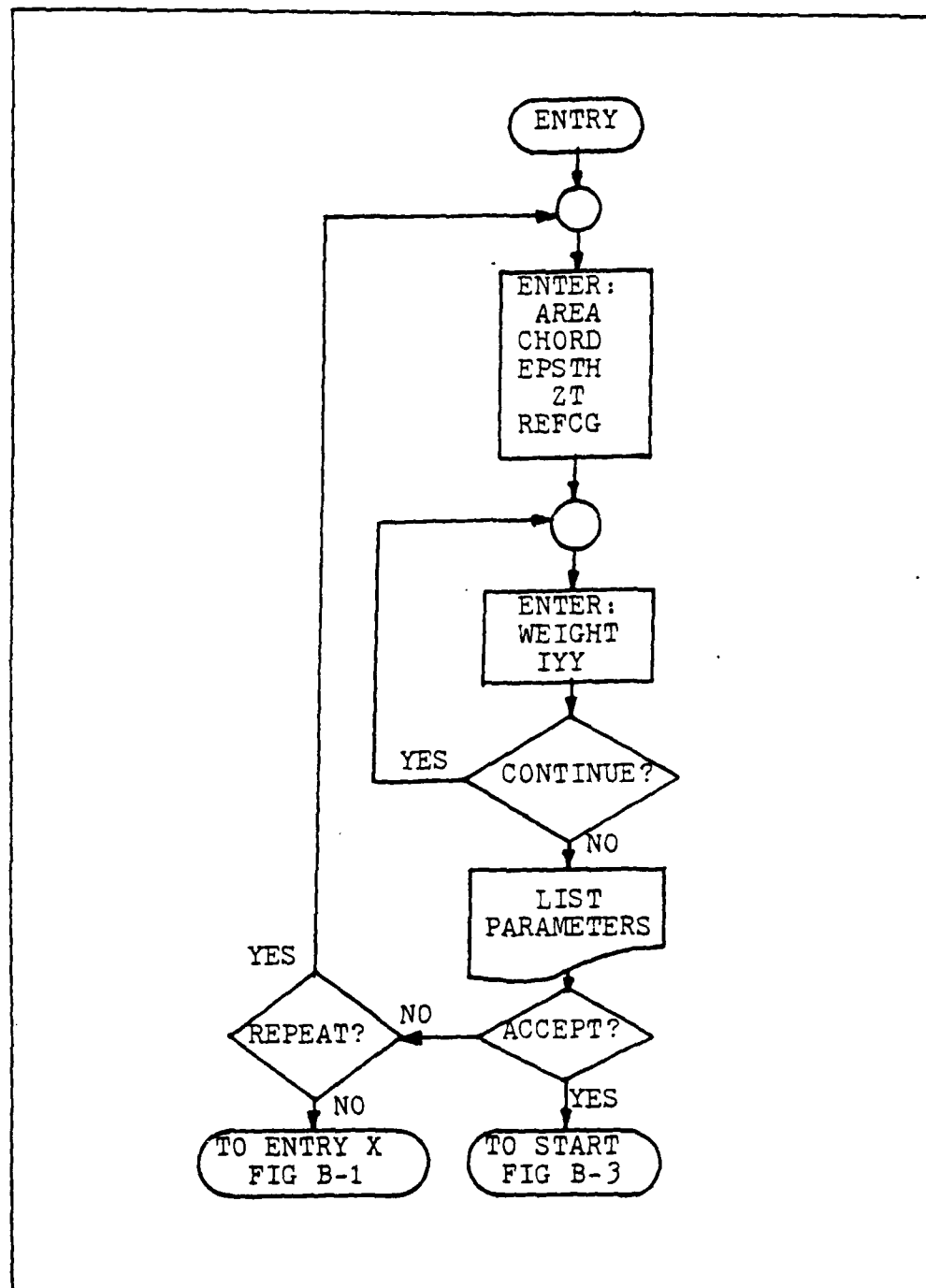


Fig B-2. Parameter Entry Logic

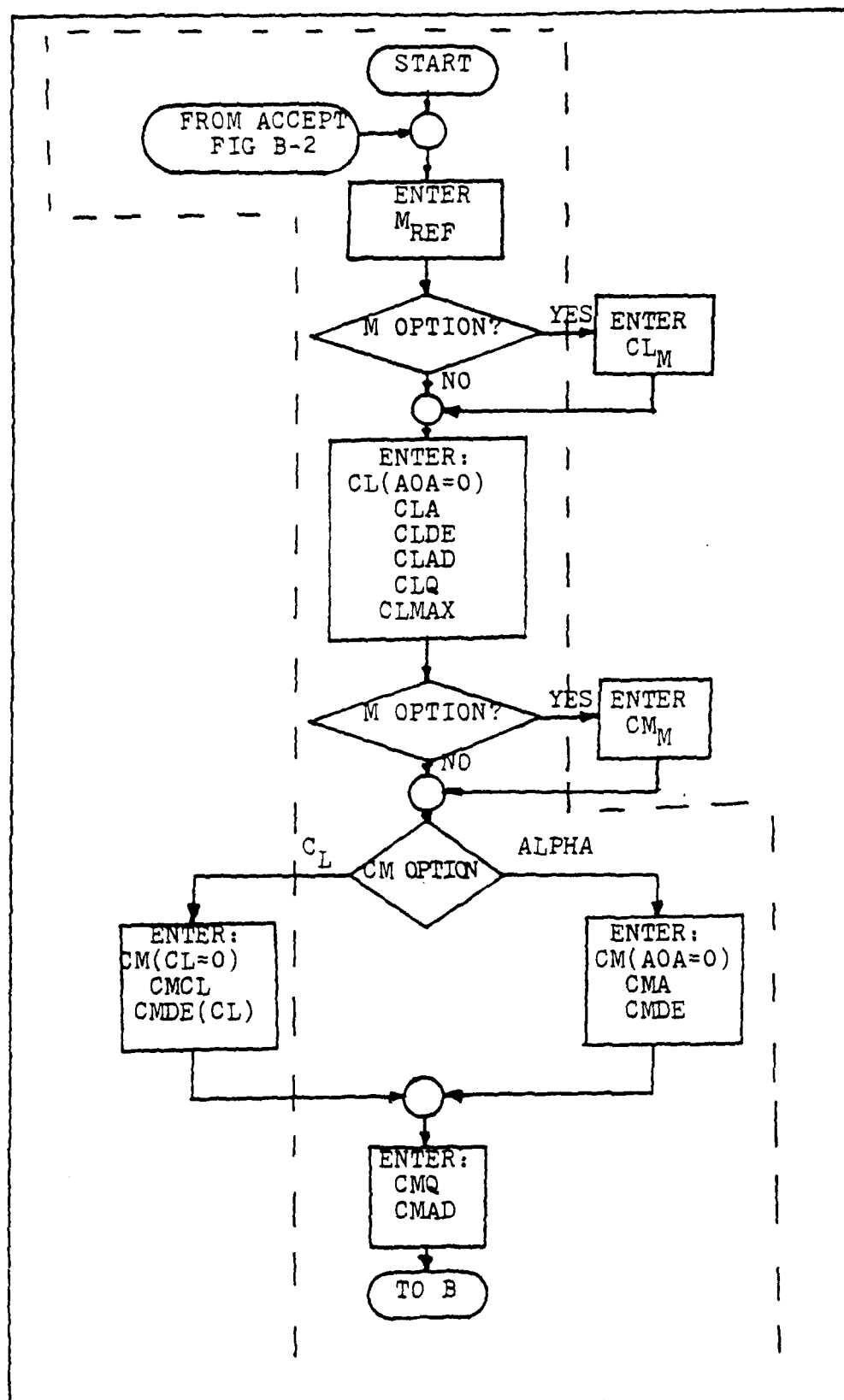


Fig B-3. Aero Data Entry Sequence



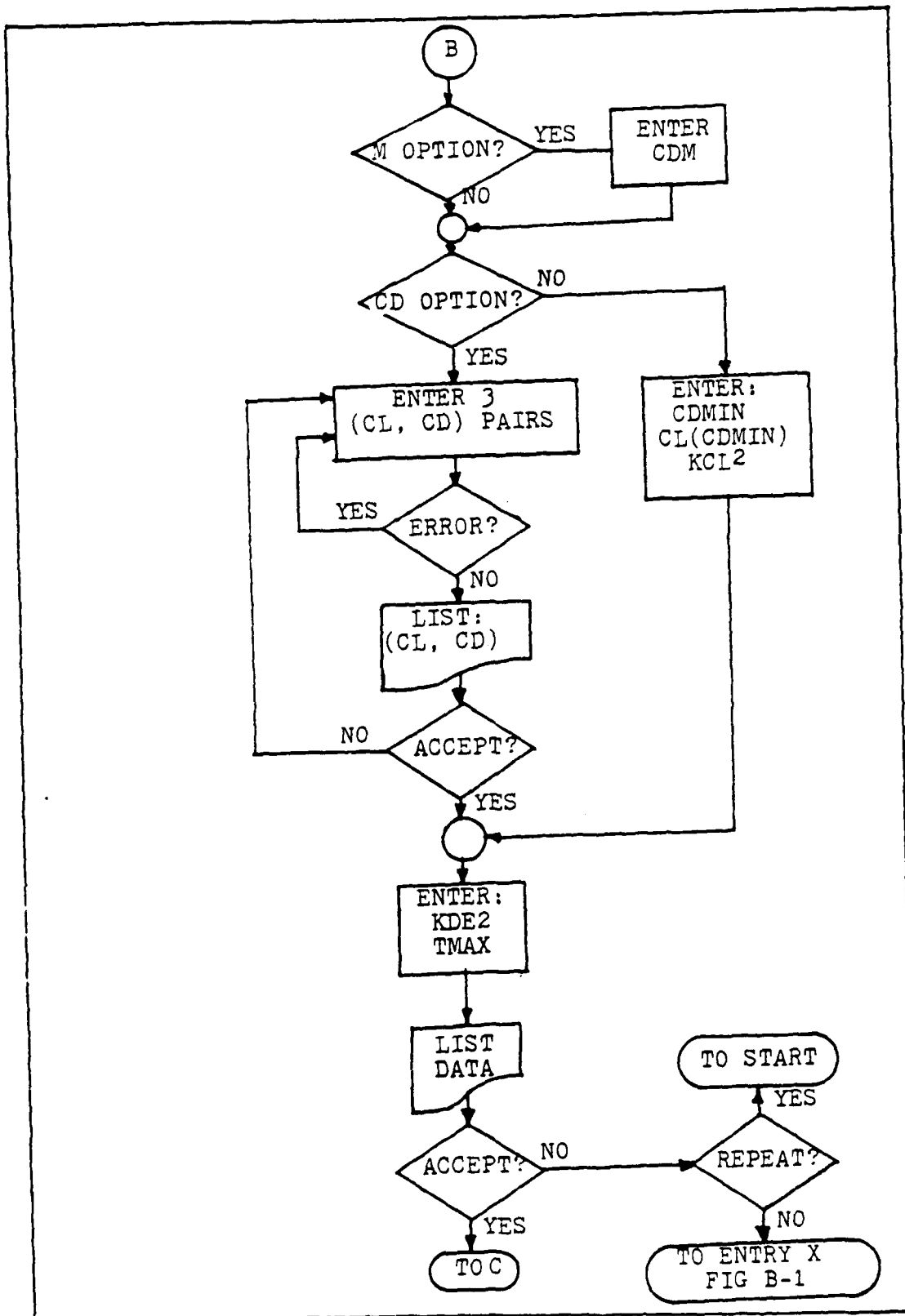


Fig B-3. Aero Data Entry Sequence (Cont'd)

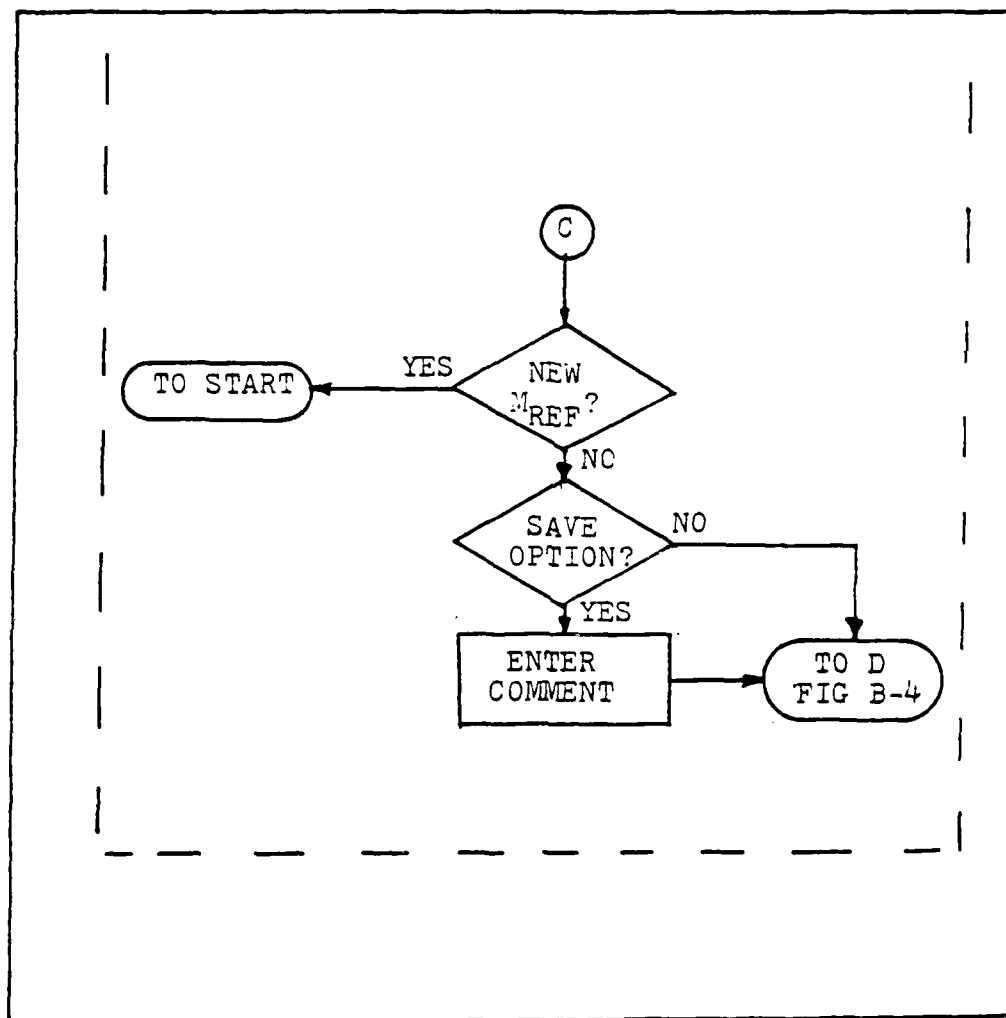


Fig B-3. Aero Data Entry Sequence (Cont'd)

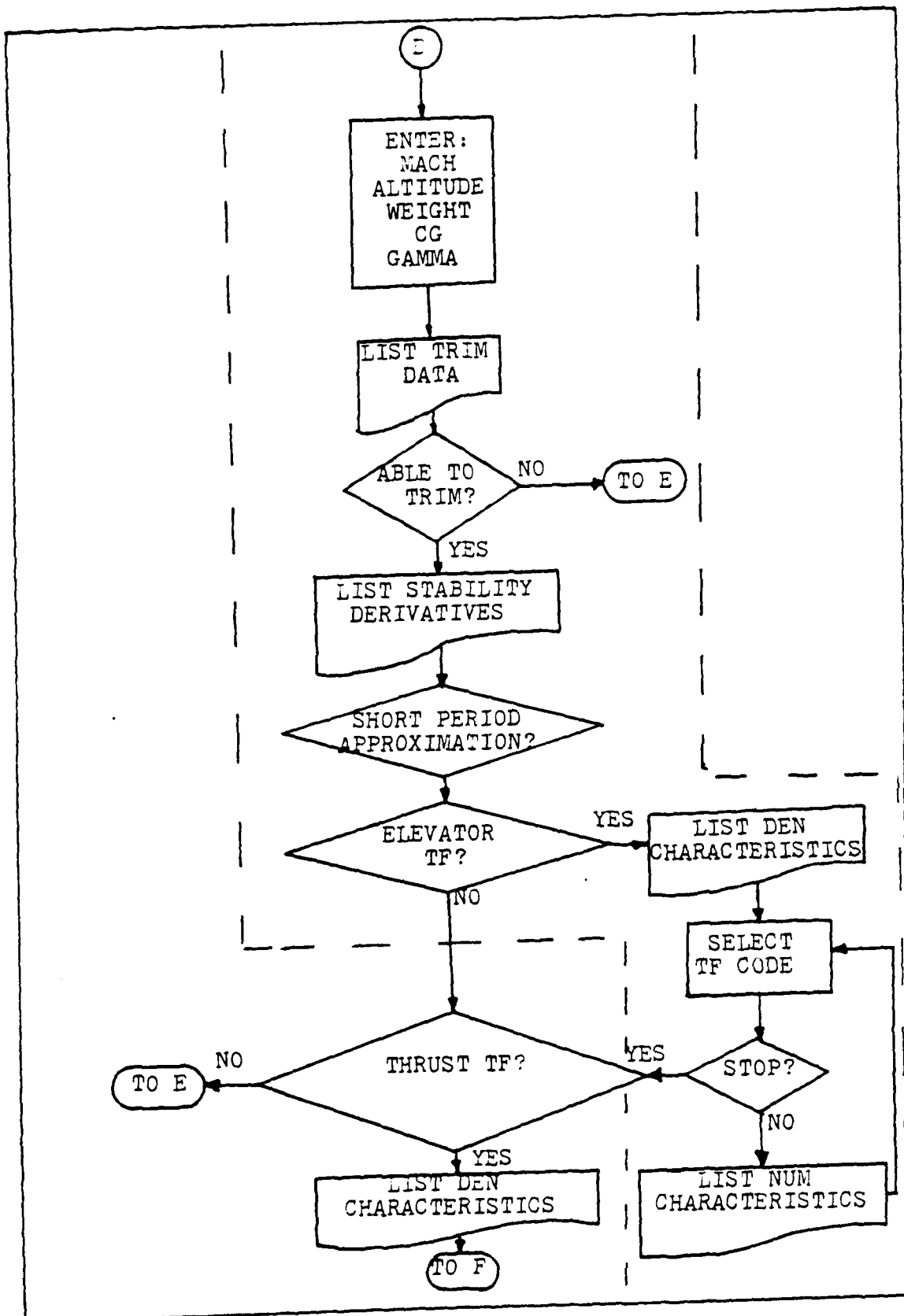


Fig B-4. Flight Case Data Section

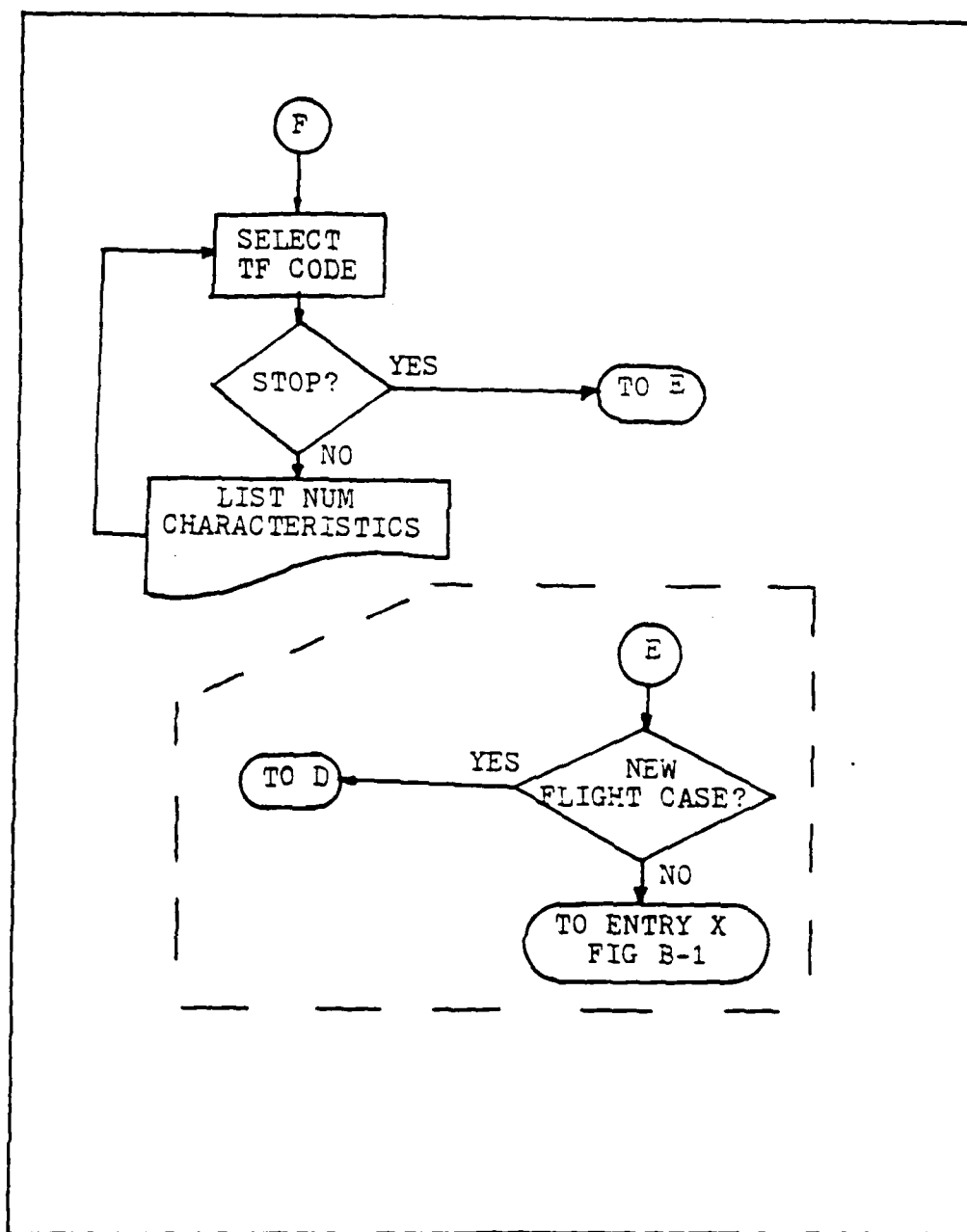


Fig B-4. Flight Case Data Section (Cont'd)

## Appendix C

### TTYLON Runs for $N_z$ Sensor Location

The location of the  $N_z$  accelerometer will determine the position of the open loop short period zeros in the  $N_z/\delta_c$  transfer function.  $L_x$ , the distance the sensor is placed behind the nose of the aircraft, has been varied from 0 ft to 50 ft. (The aircraft is 48 ft long). This causes the short period zeros to vary from a frequency of 2.8552 rad/sec to 9.1511 rad/sec. The actual TTYLON output can be seen on the following pages. A discussion of the final  $N_z$  accelerometer location choice may be found in the section, "Placement of the Normal Acceleration Sensor."

DENOMINATOR CHARACTERISTICS

POLYNOMIAL COEFFICIENTS

P(0) = -.68100E-01

P(1) = -.15726E+00

P(2) = -.11617E+02

P(3) = .12057E+01

P(4) = .10000E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.70548E-02	.76164E-01	.092	.76490E-01	.62496E+02
.28675E+01				
-.40591E+01				

TF CODES (U, AOA, Q, TH, NZ, HDOT, STOP)

TF = NZ

LX = 8

NZ/DE NUMERATOR LX = 0.000

POLYNOMIAL COEFFICIENTS

P(0) = -.31792E-02

P(1) = .23320E+00

P(2) = .03115E+02

P(3) = .11336E+01

P(4) = .98861E+00

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.77452E-02				
.49368E-02				
-.57191E+00	.91511E+01	.062	.91689E+01	.68661E+00

TF = NZ

LX = 10

NZ/DE NUMERATOR LX = 10.000

POLYNOMIAL COEFFICIENTS

P(0) = -.31792E-02

P(1) = .23320E+00

P(2) = .03129E+02

P(3) = .24923E+01

P(4) = .27930E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.77450E-02				
.49383E-02				
-.44476E+00	.54372E+01	.082	.54554E+01	.11556E+01

TF = NZ

LX = 20

NZ/DE NUMERATOR LX = 20.000

POLYNOMIAL COEFFICIENTS

P(0) = -.31792E-02

P(1) = .23320E+00

P(2) = .03143E+02

P(3) = .38510E+01

P(4) = .45974E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.77448E-02				
.49379E-02				
-.41742E+00	.42318E+01	.098	.42524E+01	.14648E+01

TF = NZ

LX = 30

NZ/DE NUMERATOR LX = 30.000

POLYNOMIAL COEFFICIENTS

P(0) = -.31792E-02

P(1) = .23320E+00

P(2) = .83157E+02

P(3) = .52097E+01

P(4) = .64018E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.77446E-02				
.49374E-02				
-.40549E+00	.35809E+01	.113	.36038E+01	.17546E+01

TF = NZ

LX = 40

NZ/DE NUMERATOR LX = 40.000

POLYNOMIAL COEFFICIENTS

P(0) = -.31792E-02

P(1) = .23320E+00

P(2) = .83171E+02

P(3) = .65684E+01

P(4) = .82061E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.77443E-02				
.49369E-02				
-.39681E+00	.31582E+01	.125	.31832E+01	.19695E+01

TF = NZ

LX = 50

NZ/DE NUMERATOR LX = 50.000

POLYNOMIAL COEFFICIENTS

P(0) = -.31792E-02

P(1) = .23320E+00

P(2) = .83165E+02

P(3) = .79271E+01

P(4) = .10011E+02

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.77441E-02				
.49364E-02				
-.39454E+00	.26552E+01	.137	.28823E+01	.22007E+01

## Appendix D

### TTYLON Runs for Flight Envelope

The transfer functions,  $N_z/\delta_c$ ,  $q/\delta_c$ , and  $\alpha/\delta_c$  were found using TTYLON. Following are the computer listings showing this output for the flight conditions given in Table I.



ENTER FLIGHT CONDITION  
MACH = .3

ALT = 0

WEIGHT = 15000

CG = 451

GAMMA = 0

# FLIGHT CASE DATA

MACH = .30000	VFPS = 335.47	VSND = 1118.22
ALT = 0.	QDYN = 133.74	DEN = .0023759
WGHT = 15000.	IYY = 47795.	AREA = 185.00
CG = 451.000	REFCG = 451.000	CHORD = 7.22
GAMMA = 0.000	EPSTH = 0.000	ZT = 0.000
AQA = 7.968	ELEV = -11.796	THRST = 1925.64

# NON-DIMENSIONAL DERIVATIVE MATRIX

PERTURBATION	CD	CL	CM
0 (PER RAD)	.07707490	.59544855	0.00000000
M (PER RAD)	.01481208	.33172654	.01855430
A (PER DEG)	.01720939	.08503333	.02901607
AD (PER RAD)	0.00000000	.40976667	-.96550000
Q (PER RAD)	0.00000000	6.75383333	-7.18366067
DE (PER DEG)	.00119177	.00588867	.01403667

# DIMENSIONAL DERIVATIVE PARAMETER MATRIX

PERTURBATION	X	Z	M
U (PER RAD)	-.31448E-02	-.94999E-01	-.25062E-02
UD (PER RAD)	-.15366E-04	.10979E-03	.16046E-04
A (PER RAD)	.15734E+02	-.26741E+03	.60975E+01
AD (PER RAD)	.36474E-01	-.26059E+00	-.38088E-01
Q (PER RAD)	.53466E+00	-.38200E+01	-.28894E+00
DE (PER RAD)	-.11069E+01	-.18236E+02	.30060E+01

USE SHORT PERIOD APPROXIMATION (YES/NO)?N

COMPUTE ELEVATOR TRANSFER FUNCTIONS (YES/NO)?Y

# DENOMINATOR CHARACTERISTICS

## POLYNOMIAL COEFFICIENTS

P(0) = -.12001E+00  
P(1) = -.14794E+00  
P(2) = -.58964E+01  
P(3) = .11347E+01  
P(4) = .10000E+01

## POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.14311E-01	.14132E+00	.101	.14204E+00	.44461E+02
.19478E+01				
-.30539E+01				

TF CODES (U,ADA,Q,TH,NZ,HDOT,STOP)

2

LX = 20

NZ/DE NUMERATOR LX = 20.000

POLYNOMIAL COEFFICIENTS

P(0) = -.36576E-02

P(1) = -.34411E+00

P(2) = .21056E+02

P(3) = .18691E+01

P(4) = .24362E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.73371E-02				
.23640E-01				
-.39156E+00	.29159E+01	.133	.29421E+01	.21548E+01

TF = Q

Q/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = 0.

P(1) = .26386E-01

P(2) = .20955E+01

P(3) = .30081E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
0.				
-.12828E-01				
-.68378E+00				

TF = AOA

AOA/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = .31582E-01

P(1) = .17535E-01

P(2) = .29533E+01

P(3) = -.54846E-01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.30677E-02	.10336E+00	.030	.10341E+00	.60790E+02
.53653E+02				

ENTER FLIGHT CONDITION  
MACH = .6

ALT = 0

WEIGHT = 15000

CG = 451

GAMMA = 0

# FLIGHT CASE DATA

MACH = .60000	VFPS = 670.93	VSND = 1116.22
ALT = 0.	GDYN = 534.98	DEN = .0023769
WGHT = 15000.	IYY = 47795.	AREA = 185.00
CG = 451.000	REFCG = 451.000	CHORD = 7.22
GAMMA = 0.000	EPSTH = 0.000	ZT = 0.000
AOA = 1.895	ELEV = .631	THRST = 2943.94

# NON-DIMENSIONAL DERIVATIVE MATRIX

PERTURBATION	CD	CL	CM
0 (PER RAD)	.02972913	.15057535	-.00000000
M (PER RAD)	.01310585	-.00563595	.00589280
A (PER DEG)	.00189691	.10439333	.03293007
AD (PER RAD)	0.00000000	.46906667	-1.00040000
Q (PER RAD)	0.00000000	7.24233333	-7.66566667
DE (PER DEG)	.00013091	.00720467	.01564007

# DIMENSIONAL DERIVATIVE PARAMETER MATRIX

PERTURBATION	X	Z	M
U (PER RAD)	-.20682E-01	-.31965E-01	-.13119E-02
UD (PER RAD)	-.87344E-06	.26396E-04	.42841E-05
A (PER RAD)	.50631E+02	-.12758E+04	.28185E+02
AD (PER RAD)	.17700E-01	-.53490E+00	-.86816E-01
Q (PER RAD)	.27359E+00	-.82679E+01	-.61666E+00
DE (PER RAD)	.13067E+01	-.87636E+02	.13403E+02

USE SHORT PERIOD APPROXIMATION (YES/NO)?N

COMPUTE ELEVATOR TRANSFER FUNCTIONS (YES/NO)?Y

# DENOMINATOR CHARACTERISTICS

## POLYNOMIAL COEFFICIENTS

P(0) = -.12255E+00  
P(1) = -.56514E+00  
P(2) = -.26615E+02  
P(3) = .26241E+01  
P(4) = .10000E+01

## POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.10817E-01	.66910E-01	.160	.67778E-01	.93906E+02
.40251E+01				
-.66276E+01				

TF CODES (U, AOA, Q, TH, NZ, HDOT, STOP)

NZ

LX = 20

NZ/DE NUMERATOR LX = 20.000

POLYNOMIAL COEFFICIENTS

P(0) = -.16101E-01

P(1) = .88829E+01

P(2) = .45438E+03

P(3) = .19359E+02

P(4) = .11060E+02

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.21237E-01				
.16699E-02				
-.86537E+00	.63482E+01	.135	.64069E+01	.98976E+00

TF = Q

Q/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = 0.

P(1) = .48683E+00

P(2) = .22075E+02

P(3) = .13415E+02

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
0.				
-.22357E-01				
-.16232E+01				

TF = AOA

AOA/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = .25601E-01

P(1) = .26663E+00

P(2) = .13144E+02

P(3) = -.13059E+00

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.10150E-01	.42945E-01	.230	.44129E-01	.14631E+03
.10068E+03				

Y  
ENTER FLIGHT CONDITION  
MACH = .8

ALT = 0

WEIGHT = 15000

CG = 451

GAMMA = 0

FLIGHT CASE DATA

MACH = .80000	VFPS = 894.58	VSND = 1118.22
ALT = 0.	QDYN = 951.08	DEN = .0023769
WGHT = 15000.	IYY = 47795.	AREA = 185.00
CG = 451.000	REFCG = 451.000	CHORD = 7.22
GAMMA = 0.000	EPSTH = 0.000	ZT = 0.000
AOA = 1.253	ELEV = 1.926	THRST = 5741.71

NON-DIMENSIONAL DERIVATIVE MATRIX

PERTURBATION	CD	CL	CM
0 (PER RAD)	.03262504	.08453863	-.00000000
M (PER RAD)	.01566414	-.04142369	.00445585
A (PER DEG)	-.00232772	.11730000	.03555000
AD (PER RAD)	0.00000000	.46860000	-1.15700000
Q (PER RAD)	0.00000000	7.56200000	-7.98700000
DE (PER DEG)	-.00016038	.00808200	.01672000

DIMENSIONAL DERIVATIVE PARAMETER MATRIX

PERTURBATION	X	Z	M
U (PER RAD)	-.34916E-01	.41841E-02	-.12170E-02
UD (PER RAD)	-.38122E-06	.17435E-04	.30325E-05
A (PER RAD)	.13725E+03	-.25462E+04	.54114E+02
AD (PER RAD)	.15593E-01	-.71315E+00	-.12404E+00
Q (PER RAD)	.25196E+00	-.11523E+02	-.85667E+00
DE (PER RAD)	.72874E+01	-.17464E+03	.25462E+02

USE SHORT PERIOD APPROXIMATION (YES/NO)?N

COMPUTE ELEVATOR TRANSFER FUNCTIONS (YES/NO)?Y

DENOMINATOR CHARACTERISTICS

POLYNOMIAL COEFFICIENTS

P(0) = -.10188E+00

P(1) = -.16726E+01

P(2) = -.50828E+02

P(3) = .38587E+01

P(4) = .10000E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.16487E-01	.41562E-01	.369	.44713E-01	.15118E+03
.54774E+01				
-.93831E+01				

TF CODES (U, AOA, Q, TH, NZ, HDOT, STOP)

NZ

LX = 20

NZ/DE NUMERATOR LX = 20.000

POLYNOMIAL COEFFICIENTS

P(0) = -.47071E-01

P(1) = .58398E+02

P(2) = .17218E+04

P(3) = .54195E+02

P(4) = .21267E+02

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.34741E-01				
.78775E-03				
-.12572E+01	.89048E+01	.140	.89931E+01	.70560E+00

TF = Q

Q/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = 0.

P(1) = .21533E+01

P(2) = .62756E+02

P(3) = .25487E+02

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
0.				
-.34804E-01				
-.24275E+01				

TF = AQA

AQA/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = .31181E-02

P(1) = .84457E+00

P(2) = .24940E+02

P(3) = -.19511E+00

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.29638E-01				
-.42172E-02				
.12786E+03				

ENTER FLIGHT CONDITION  
MACH = .4

ALT = 20000

WEIGHT = 15000

CG = 451

GAMMA = 0

FLIGHT CASE DATA

MACH = .40000	VFPS = 415.40	VSND = 1038.50
ALT = 20000.	QDYN = 109.18	DEN = .0012655
WGHT = 15000.	IYY = 47795.	AREA = 185.00
CG = 451.000	REFCG = 451.000	CHORD = 7.22
GAMMA = 0.000	EPSTH = 0.000	ZT = 0.000
AOA = 9.582	ELEV = -14.768	THRST = 2223.59

NON-DIMENSIONAL DERIVATIVE MATRIX

PERTURBATION	CD	CL	CM
0 (PER RAD)	.10855090	.72430230	-.00000000
M (PER RAD)	.02146685	.28583536	.00047312
A (PER DEG)	.02214529	.08547333	.02680667
AD (PER RAD)	0.00000000	.46953333	-1.00380000
Q (PER RAD)	0.00000000	6.91666667	-7.34433333
DE (PER DEG)	.00140297	.00541500	.01401667

DIMENSIONAL DERIVATIVE PARAMETER MATRIX

PERTURBATION	X	Z	M
U (PER RAD)	-.12750E-02	-.75087E-01	-.20167E-02
UD (PER RAD)	-.11792E-04	.69854E-04	.10666E-04
A (PER RAD)	.12751E+02	-.22297E+03	.48966E+01
AD (PER RAD)	.28612E-01	-.16949E+00	-.25880E-01
Q (PER RAD)	.43350E+00	-.25679E+01	-.19475E+00
DE (PER RAD)	-.11965E+01	-.13834E+02	.24504E+01

USE SHORT PERIOD APPROXIMATION (YES/NO)?N

COMPUTE ELEVATOR TRANSFER FUNCTIONS (YES/NO)?Y

DENOMINATOR CHARACTERISTICS

POLYNOMIAL COEFFICIENTS

P(0) = -.63533E-01  
P(1) = -.11674E+00  
P(2) = -.48925E+01  
P(3) = .76659E+00  
P(4) = .10000E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.12824E-01	.11286E+00	.113	.11359E+00	.55673E+02
.18793E+01				
-.26203E+01				

TF CODES (U, AOA, Q, TH, NZ, HDOT, STOP)

NZ

LX = 20

NZ/DE NUMERATOR LX = 20.000

POLYNOMIAL COEFFICIENTS

P(0) = -.17427E-02

P(1) = -.27000E+00

P(2) = .14424E+02

P(3) = .10312E+01

P(4) = .19536E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.50776E-02				
.23761E-01				
-.27326E+00	.27054E+01	.100	.27192E+01	.23225E+01

TF = 0

Q/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = 0.

P(1) = .10469E-01

P(2) = .11736E+01

P(3) = .24513E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
0.				
-.90935E-02				
-.46966E+00				

TF = AOA

AOA/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = .16333E-01

P(1) = .90149E-02

P(2) = .24276E+01

P(3) = -.33760E-01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.19034E-02	.81999E-01	.023	.82021E-01	.76625E+02
.71911E+02				



Y  
ENTER FLIGHT CONDITION

MACH = .6

ALT = 20000

WEIGHT = 15000

CG = 451

GAMMA = 0

FLIGHT CASE DATA

MACH = .60000	VFPS = 623.10	USND = 1038.50
ALT = 20000.	QDYN = 245.66	DEN = .0012655
WGHT = 15000.	IYY = 47795.	AREA = 185.00
CG = 451.000	REFCG = 451.000	CHORD = 7.22
GAMMA = 0.000	EPSTH = 0.000	ZT = 0.000
AOA = 4.035	ELEV = -3.333	THRST = 1814.74

NON-DIMENSIONAL DERIVATIVE MATRIX

PERTURBATION	CD	CL	CM
0 (PER RAD)	.03983219	.32724656	-.00000000
M (PER RAD)	.00983481	.07557654	-.00251972
A (PER DEG)	.00900309	.09352667	.03043333
AD (PER RAD)	0.00000000	.46906667	-1.05040000
Q (PER RAD)	0.00000000	7.24233333	-7.66566667
DE (PER DEG)	.00054575	.00567300	.01475333

DIMENSIONAL DERIVATIVE PARAMETER MATRIX

PERTURBATION	X	Z	M
U (PER RAD)	-.77483E-02	-.50411E-01	-.13636E-02
UD (PER RAD)	-.21056E-05	.29846E-04	.48534E-05
A (PER RAD)	.18380E+02	-.52878E+03	.11911E+02
AD (PER RAD)	.18551E-01	-.26295E+00	-.42760E-01
Q (PER RAD)	.26784E+00	-.40801E+01	-.30490E+00
DE (PER RAD)	-.81075E+00	-.31921E+02	.58032E+01

USE SHORT PERIOD APPROXIMATION (YES/NO)?N

COMPUTE ELEVATOR TRANSFER FUNCTIONS (YES/NO)?Y

DENOMINATOR CHARACTERISTICS

POLYNOMIAL COEFFICIENTS

P(0) = -.68100E-01  
P(1) = -.15726E+00  
P(2) = -.11617E+02  
P(3) = .12057E+01  
P(4) = .10000E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.70548E-02	.76164E-01	.092	.76490E-01	.82496E+02
.23675E+01				
-.40591E+01				

TF CODES (U, AOA, Q, TH, NZ, HDOT, STOP)

NZ

LX = 20

NZ/DE NUMERATOR LX = 20.000

POLYNOMIAL COEFFICIENTS

P(0) = -.31792E-02

P(1) = .23320E+00

P(2) = .83143E+02

P(3) = .38510E+01

P(4) = .45974E+01

POLYNOMIAL FACTORS

REAL PART

IMAG PARTS

DR

WN

PERIOD

-.77448E-02

.49379E-02

-.41742E+00

.42318E+01

.098

.42524E+01

.14848E+01

TF = Q

Q/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = 0.

P(1) = .45175E-01

P(2) = .43715E+01

P(3) = .58854E+01

POLYNOMIAL FACTORS

REAL PART

IMAG PARTS

DR

WN

PERIOD

0.

-.10480E-01

-.74253E+00

TF = AOA

AOA/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = .17180E-01

P(1) = .48850E-01

P(2) = .57467E+01

P(3) = -.51174E-01

POLYNOMIAL FACTORS

REAL PART

IMAG PARTS

DR

WN

PERIOD

-.41936E-02

.11231E+03

.54513E-01

.077

.54674E-01

.11526E+03

Y  
ENTER FLIGHT CONDITION  
MACH = 1

ALT = 20000

WEIGHT = 15000

CG = 451

GAMMA = 0

FLIGHT CASE DATA

MACH = 1.00000	VFPS = 1038.50	VSND = 1038.50
ALT = 20000.	QDYN = 682.38	DEN = .0012655
WGHT = 15000.	IYY = 47795.	AREA = 185.00
CG = 451.000	REFCG = 451.000	CHORD = 7.22
GAMMA = 0.000	EPSTH = 0.000	ZT = 0.000
AOA = 1.719	ELEV = .748	THRST = 4448.03

NON-DIMENSIONAL DERIVATIVE MATRIX

PERTURBATION	CD	CL	CM
0 (PER RAD)	.03521861	.11776344	0.00000000
M (PER RAD)	.01495357	-.06609252	.05778858
A (PER DEG)	-.00092374	.09711750	.02259650
AD (PER RAD)	0.00000000	.45172500	-1.09200000
Q (PER RAD)	0.00000000	6.83175000	-7.62075000
DE (PER DEG)	-.00009914	.01042325	.01307525

DIMENSIONAL DERIVATIVE PARAMETER MATRIX

PERTURBATION	X	Z	M
U (PER RAD)	-.23569E-01	-.10673E-02	.34752E-03
UD (PER RAD)	-.36844E-06	.12277E-04	.20911E-05
A (PER RAD)	.90949E+02	-.15142E+04	.24701E+02
AD (PER RAD)	.12744E-01	-.42462E+00	-.72325E-01
Q (PER RAD)	.19290E+00	-.64277E+01	-.50519E+00
DE (PER RAD)	.63884E+01	-.16159E+03	.14287E+02

USE SHORT PERIOD APPROXIMATION (YES/NO)?N

COMPUTE ELEVATOR TRANSFER FUNCTIONS (YES/NO)?Y

DENOMINATOR CHARACTERISTICS

POLYNOMIAL COEFFICIENTS

P(0) = .16050E-01  
P(1) = -.54316E+00  
P(2) = -.23742E+02  
P(3) = .20588E+01  
P(4) = .10000E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.39750E-01				
.16974E-01				
.39644E+01				
-.60004E+01				

TF CODES (U, AOA, Q, TH, NZ, HOOT, STOP)

NZ

LX = 20

NZ/DE NUMERATOR LX = 20.000

POLYNOMIAL COEFFICIENTS

P(0) = -.11996E-01

P(1) = .12433E+02

P(2) = .54852E+03

P(3) = .16631E+02

P(4) = .13908E+02

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.23609E-01				
.92694E-03				
-.59373E+00	.62498E+01	.095	.62779E+01	.10054E+01

TF = Q

Q/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = 0.

P(1) = .39988E+00

P(2) = .17328E+02

P(3) = .14298E+02

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
0.				
-.23535E-01				
-.11884E+01				

TF = AOA

AOA/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = -.15819E-02

P(1) = .32033E+00

P(2) = .14110E+02

P(3) = -.15561E+00

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.26867E-01				
.41717E-02				
.90699E+02				

ENTER FLIGHT CONDITION  
MACH = 1.2

ALT = 20000

WEIGHT = 15000

CG = 451

GAMMA = 0

# FLIGHT CASE DATA

MACH = 1.20000	VFPS = 1246.20	VSND = 1038.50
ALT = 20000.	QDYN = 982.63	DEN = .0012655
WGHT = 15000.	IYY = 47795.	AREA = 165.00
CG = 451.000	REFCG = 451.000	CHORD = 7.22
GAMMA = 0.000	EPSTH = 0.000	ZT = 0.000
AOA = 1.625	ELEV = -.220	THRST = 7126.20

# NON-DIMENSIONAL DERIVATIVE MATRIX

PERTURBATION	CD	CL	CM
0 (PER RAD)	.03918507	.08140251	0.00000000
M (PER RAD)	.01623864	-.08568697	.07392841
A (PER DEG)	-.00358194	.09253500	.01313500
AD (PER RAD)	0.00000000	.43485000	-1.02700000
Q (PER RAD)	0.00000000	6.09550000	-7.25450000
DE (PER DEG)	-.00057736	.01491550	.01006050

# DIMENSIONAL DERIVATIVE PARAMETER MATRIX

PERTURBATION	X	Z	M
U (PER RAD)	-.33948E-01	.27680E-01	.14839E-02
UD (PER RAD)	-.31696E-06	.11173E-04	.18591E-05
A (PER RAD)	.16958E+03	-.20776E+04	.20716E+02
AD (PER RAD)	.13918E-01	-.49058E+00	-.81632E-01
Q (PER RAD)	.19525E+00	-.68923E+01	-.57709E+00
DE (PER RAD)	.22343E+02	-.33272E+03	.16773E+02

USE SHORT PERIOD APPROXIMATION (YES/NO)?N

COMPUTE ELEVATOR TRANSFER FUNCTIONS (YES/NO)?Y

# DENOMINATOR CHARACTERISTICS

## POLYNOMIAL COEFFICIENTS

P(0) = .95061E-01  
P(1) = -.75271E+00  
P(2) = -.19504E+02  
P(3) = .23594E+01  
P(4) = .10000E+01

## POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.91117E-01				
.53264E-01				
.34147E+01				
-.57362E+01				

TF CODES (U,AOA,Q,TH,NZ,HDOT,STOP)

NZ

LX = 20

NZ/DE NUMERATOR LX = 20.000

POLYNOMIAL COEFFICIENTS

P(0) = -.19750E-01

P(1) = .27297E+02

P(2) = .87030E+03

P(3) = .25304E+02

P(4) = .20776E+02

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.32099E-01				
.70704E-03				
-.59323E+00	.64421E+01	.092	.64694E+01	.97503E+00

TF = Q

Q/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = 0.

P(1) = .69674E+00

P(2) = .23036E+02

P(3) = .16795E+02

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
0.				
-.30944E-01				
-.13406E+01				

TF = AQA

AQA/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = -.25165E-01

P(1) = .55474E+00

P(2) = .16511E+02

P(3) = -.26699E+00

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.59260E-01				
.25705E-01				
.61875E+02				

ENTER FLIGHT CONDITION  
MACH = .5

ALT = 40000

WEIGHT = 15000

CG = 451

GAMMA = 0

FLIGHT CASE DATA

MACH =	.50000	UFPS =	484.81	USND =	969.62
ALT =	40000.	QDYN =	68.70	DEN =	.0005245
WGHT =	15000.	IYY =	47795.	AREA =	185.00
CG =	451.000	REFCG =	451.000	CHORD =	7.22
GAMMA =	0.000	EPSTH =	0.000	ZT =	0.000
AOA =	14.537	ELEV =	-24.827	THRST =	3246.79

NON-DIMENSIONAL DERIVATIVE MATRIX

PERTURBATION	CD	CL	CM
0 (PER RAD)	.24729560	1.11615331	0.00000000
M (PER RAD)	.05870757	.37327587	-.01916400
A (PER DEG)	.03742975	.08667000	.02878000
AD (PER RAD)	0.00000000	.46930000	-1.04210000
Q (PER RAD)	0.00000000	7.07950000	-7.50500000
DE (PER DEG)	.00218394	.00505700	.01393000

DIMENSIONAL DERIVATIVE PARAMETER MATRIX

PERTURBATION	X	Z	M
U (PER RAD)	.10245E-02	-.59739E-01	-.16757E-02
UD (PER RAD)	-.12379E-04	.47740E-04	.77129E-05
A (PER RAD)	.86572E+01	-.15186E+03	.29618E+01
AD (PER RAD)	.22404E-01	-.86401E-01	-.13959E-01
Q (PER RAD)	.36069E+00	-.13910E+01	-.10729E+00
DE (PER RAD)	-.13193E+01	-.85017E+01	.15323E+01

USE SHORT PERIOD APPROXIMATION (YES/NO)?N

COMPUTE ELEVATOR TRANSFER FUNCTIONS (YES/NO)?Y

DENOMINATOR CHARACTERISTICS

POLYNOMIAL COEFFICIENTS

P(0) = -.28927E-01  
P(1) = -.95033E-01  
P(2) = -.31199E+01  
P(3) = .44466E+00  
P(4) = .10000E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.15728E-01	.94651E-01	.164	.95949E-01	.66383E+02
.15780E+01				
-.19912E+01				

TF CODES (U, AOA, Q, TH, NZ, HDOT, STOP)

NZ

LX = 20

NZ/DE NUMERATOR LX = 20.000

POLYNOMIAL COEFFICIENTS

P(0) = -.68040E-03

P(1) = -.17381E+00

P(2) = .60512E+01

P(3) = .37541E+00

P(4) = .12168E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.34906E-02				
.32149E-01				
-.16858E+00	.22258E+01	.076	.22322E+01	.28229E+01

TF = Q

Q/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = 0.

P(1) = .27108E-02

P(2) = .44275E+00

P(3) = .15325E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
0.				
-.62581E-02				
-.28265E+00				

TF = AOA

AOA/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = .70078E-02

P(1) = .16300E-02

P(2) = .15256E+01

P(3) = -.18113E-01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.56145E-03	.67771E-01	.008	.67774E-01	.92712E+02
.84231E+02				



ENTER FLIGHT CONDITION  
MACH = .6

ALT = 40000

WEIGHT = 15000

CG = 451

GAMMA = 0

# FLIGHT CASE DATA

MACH = .60000	VFPS = 581.77	USNO = 969.62
ALT = 40000.	QDYN = 98.92	DEN = .0005546
WGHT = 15000.	IYY = 47795.	AREA = 185.00
CG = 451.000	REFCG = 451.000	CHORD = 7.22
GAMMA = 0.000	EPSTH = 0.000	ZT = 0.000
AOA = 10.019	ELEV = -15.382	THRST = 2480.39

# NON-DIMENSIONAL DERIVATIVE MATRIX

PERTURBATION	CD	CL	CM
0 (PER RAD)	.13346745	.79605687	0.00000000
M (PER RAD)	.02637223	.23666727	-.01553995
A (PER DEG)	.02722351	.08977000	.02939667
AD (PER RAD)	0.00000000	.46906667	-1.08040000
Q (PER RAD)	0.00000000	7.24233333	-7.66566667
DE (PER DEG)	.00154460	.00509333	.01426333

# DIMENSIONAL DERIVATIVE PARAMETER MATRIX

PERTURBATION	X	Z	M
U (PER RAD)	-.40334E-03	-.54172E-01	-.14361E-02
UD (PER RAD)	-.59446E-05	.33647E-04	.55426E-05
A (PER RAD)	.64750E+01	-.21468E+03	.45110E+01
AD (PER RAD)	.19276E-01	-.10911E+00	-.17973E-01
Q (PER RAD)	.30692E+00	-.17372E+01	-.13150E+00
DE (PER RAD)	-.14279E+01	-.11885E+02	.22593E+01

USE SHORT PERIOD APPROXIMATION (YES/NO)?N

COMPUTE ELEVATOR TRANSFER FUNCTIONS (YES/NO)?Y

# DENOMINATOR CHARACTERISTICS

## POLYNOMIAL COEFFICIENTS

P(0) = -.38632E-01  
P(1) = -.91079E-01  
P(2) = -.45909E+01  
P(3) = .52503E+00  
P(4) = .10000E+01

## POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.10247E-01	.80888E-01	.126	.81534E-01	.77678E+02
.19091E+01				
-.24136E+01				

TF CODES (U, AOA, Q, TH, NZ, HDOT, STOP)

NZ

LX = 20

NZ/DE NUMERATOR LX = 20.000

POLYNOMIAL COEFFICIENTS

P(0) = -.56662E-03

P(1) = -.21030E+00

P(2) = .12971E+02

P(3) = .65206E+00

P(4) = .17740E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.23529E-02				
.18550E-01				
-.19211E+00	.26983E+01	.071	.27052E+01	.33286E+01

TF = Q

Q/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = 0.

P(1) = .32568E-02

P(2) = .75584E+00

P(3) = .22596E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
0.				
-.43659E-02				
-.33013E+00				

TF = AOA

AOA/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = .76821E-02

P(1) = .54500E-02

P(2) = .22494E+01

P(3) = -.20742E-01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.12272E-02	.58427E-01	.021	.58439E-01	.10754E+03
.10845E+03				

Y  
ENTER FLIGHT CONDITION

MACH = 1

ALT = 40000

WEIGHT = 15000

CG = 451

GAMMA = 0

FLIGHT CASE DATA

MACH = 1.00000	VFPS = 969.62	VSND = 969.62
ALT = 40000.	QDYN = 274.79	DEN = .0005846
WGHT = 15000.	IYY = 47795.	AREA = 185.00
CG = 451.000	REFCG = 451.000	CHORD = 7.22
GAMMA = 0.000	EPSTH = 0.000	ZT = 0.000
AOA = 3.873	ELEV = -1.969	THRST = 2147.24

NON-DIMENSIONAL DERIVATIVE MATRIX

PERTURBATION	CD	CL	CM
0 (PER RAD)	.04214259	.29221647	-.00000000
M (PER RAD)	.01697672	-.15295772	-.04446168
A (PER DEG)	.00819174	.09215500	.02202400
AD (PER RAD)	0.00000000	.45172500	-1.09200000
Q (PER RAD)	0.00000000	6.83175000	-7.62075000
DE (PER DEG)	.00072353	.00813950	.01293650

DIMENSIONAL DERIVATIVE PARAMETER MATRIX

PERTURBATION	X	Z	M
U (PER RAD)	-.94538E-02	-.86439E-02	-.10264E-02
UD (PER RAD)	-.86282E-06	.12745E-04	.21749E-05
A (PER RAD)	.19307E+02	-.58088E+03	.96232E+01
AD (PER RAD)	.12330E-01	-.18213E+00	-.31079E-01
Q (PER RAD)	.18733E+00	-.27671E+01	-.21788E+00
DE (PER RAD)	-.10753E+01	-.51040E+02	.57140E+01

USE SHORT PERIOD APPROXIMATION (YES/NO)?N

COMPUTE ELEVATOR TRANSFER FUNCTIONS (YES/NO)?Y

DENOMINATOR CHARACTERISTICS

POLYNOMIAL COEFFICIENTS

P(0) = -.22379E-01

P(1) = -.12680E+00

P(2) = -.95218E+01

P(3) = .85880E+00

P(4) = .10000E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.67527E-02	.47972E-01	.139	.48445E-01	.13098E+03
.26941E+01				
-.35394E+01				

TF CODES (U, AOA, Q, TH, NZ, HDOT, STOP)

NZ

LX = 20

NZ/DE NUMERATOR LX = 20.000

POLYNOMIAL COEFFICIENTS

P(0) = -.20544E-02

P(1) = .61561E+00

P(2) = .87699E+02

P(3) = .27847E+01

P(4) = .51390E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.94906E-02				
.24688E-02				
-.26742E+00	.41219E+01	.065	.41306E+01	.15243E+01

TF = 0

Q/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = 0.

P(1) = .30416E-01

P(2) = .29778E+01

P(3) = .57156E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
0.				
-.10423E-01				
-.51057E+00				

TF = AOA

AOA/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = .32527E-02

P(1) = .48877E-01

P(2) = .56845E+01

P(3) = -.52750E-01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.43014E-02	.23530E-01	.180	.23920E-01	.26703E+03
.10777E+03				

ENTER FLIGHT CONDITION  
MACH = 1.4

ALT = 40000

WEIGHT = 15000

CG = 451

GAMMA = 0

# FLIGHT CASE DATA

MACH = 1.40000	VFPS = 1357.46	VSND = 969.62
ALT = 40000.	QDYN = 538.58	DEN = .0005846
WGHT = 15000.	IYY = 47795.	AREA = 185.00
CG = 451.000	REFCG = 451.000	CHORD = 7.22
GAMMA = 0.000	EPSTH = 0.000	ZT = 0.000
AOA = 2.449	ELEV = .739	THRST = 4081.30

# NON-DIMENSIONAL DERIVATIVE MATRIX

PERTURBATION	CD	CL	CM
0 (PER RAD)	.04092404	.14279555	-.00000000
M (PER RAD)	.01432249	-.08553634	-.00949051
A (PER DEG)	.00022160	.08452500	.00481200
AD (PER RAD)	0.00000000	.41797500	-.96200000
Q (PER RAD)	0.00000000	5.35925000	-6.88825000
DE (PER DTG)	.00003693	.01408650	.00909950

# DIMENSIONAL DERIVATIVE PARAMETER MATRIX

PERTURBATION	X	Z	M
U (PER RAD)	-.17137E-01	.41495E-02	-.27780E-03
UD (PER RAD)	-.31944E-06	.74700E-05	.12119E-05
A (PER RAD)	.72660E+02	-.10413E+04	.41337E+01
AD (PER RAD)	.10131E-01	-.23691E+00	-.38436E-01
Q (PER RAD)	.13014E+00	-.30431E+01	-.27572E+00
DE (PER RAD)	.69177E+01	-.17235E+03	.78473E+01

USE SHORT PERIOD APPROXIMATION (YES/NO)?

N

COMPUTE ELEVATOR TRANSFER FUNCTIONS (YES/NO)?Y

# DENOMINATOR CHARACTERISTICS

## POLYNOMIAL COEFFICIENTS

P(0) = -.63973E-02  
P(1) = -.63287E-01  
P(2) = -.39098E+01  
P(3) = .10989E+01  
P(4) = .10000E+01

## POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.82773E-02	.39491E-01	.205	.40349E-01	.15910E+03
.15136E+01				
-.25960E+01				

TF CODES (U, AOA, Q, TH, NZ, HDOT, STOP)

NZ

LX = 20

NZ/DE NUMERATOR LX = 20.000

POLYNOMIAL COEFFICIENTS

P(0) = -.40018E-02

P(1) = .36980E+01

P(2) = .23187E+03

P(3) = .60745E+01

P(4) = .10237E+02

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.16973E-01				
.10173E-02				
-.28872E+00	.47495E+01	.061	.47582E+01	.13229E+01

TF = Q

Q/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = 0.

P(1) = .93665E-01

P(2) = .56315E+01

P(3) = .78522E+01

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
0.				
-.17037E-01				
-.70016E+00				

TF = AOA

AOA/DE NUMERATOR

POLYNOMIAL COEFFICIENTS

P(0) = .22863E-03

P(1) = .12433E+00

P(2) = .77911E+01

P(3) = -.12706E+00

POLYNOMIAL FACTORS

REAL PART	IMAG PARTS	DR	WN	PERIOD
-.13834E-01				
-.21207E-02				
.61334E+02				

## Appendix E

### Root Loci

The root locus design method was the frequency domain technique used. The computer program TOTAL, which is maintained by AFIT/ENE at Wright-Patterson AFB, was used to get listings and plots of root loci. First the open loop  $N_z/\delta_c$  root loci were listed for each flight condition. Following are these listings with only the short period branch listed. One can see that the damping ratio (zeta) and gain at each point in the locus are listed. As is explained in the section, "Selection of Gains,"  $K_{N_z} = -1.2$  was chosen. Following the open loop  $N_z$  root loci are the listings of the roots which result from the  $K_{N_z} = -1.2$ . Option 42 of TOTAL was used to get these results.

Once the  $N_z$  loop was closed, the roots of  $q/\delta_c$  were found with the desired damping ratio, 0.7. Option 43 of TOTAL was used to get these results. Finally, Option 42 was used to get the roots of  $q/\delta_c$  which resulted from the gain schedules for  $K_q$  in Methods I, II, and III.

GAIN=-1,41

M = 0.3  
h = 0 ft

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 41

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

REGION OF CALCULATION-REAL: CC= -15.0 TO AA= 10.0

IMAJ: DD= -.100E-01 TO BB= 15.0

BRANCH STARTING AT (-.014311) + J(.14132)  
TYPE L TO LIST, S TO SKIP, OR \$ TO ABORT >S

BRANCH STARTING AT (-.014311) + J(-.14132)  
TYPE L TO LIST, S TO SKIP, OR \$ TO ABORT >S

BRANCH STARTING AT (1.9478) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \$ TO ABORT >S

BRANCH STARTING AT (-3.0539) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \$ TO ABORT >L

BRANCH NUMBER 4

CALCULATION STEP SIZE = .1250

PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
-3.0539000	0.	3.0539000	0.	1.00000	1
-3.6789000	0.	3.6789000	.753159E-01	1.00000	0
-4.3039000	0.	4.3039000	.135557	1.00000	0
-4.9289000	0.	4.9289000	.192692	1.00000	0
-5.5539000	0.	5.5539000	.219992	1.00000	0
-6.1789000	0.	6.1789000	.249224	1.00000	0
-6.8039000	0.	6.8039000	.272465	1.00000	0
-7.4289000	0.	7.4289000	.291140	1.00000	0
-8.0539000	0.	8.0539000	.306311	1.00000	0
-8.6789000	0.	8.6789000	.318770	1.00000	0
-9.3039000	0.	9.3039000	.329107	1.00000	0
-9.9289000	0.	9.9289000	.337766	1.00000	0
-10.553900	0.	10.553900	.345086	1.00000	0
-11.178900	0.	11.178900	.351326	1.00000	0
-11.803900	0.	11.803900	.356685	1.00000	0
-12.428900	0.	12.428900	.361321	1.00000	0
-13.053900	0.	13.053900	.365359	1.00000	0
-13.678900	0.	13.678900	.368896	1.00000	0
-14.303900	0.	14.303900	.372012	1.00000	0
-14.928900	0.	14.928900	.374771	1.00000	0
-15.053900	0.	15.053900	.375285	1.00000	0

BOUNDARY

SUB-BRANCH STARTING AT X = -.22949E-01 Y = -.10000E-01

CALCULATION STEP SIZE = .1250

PRINTING STEP SIZE = .250



LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
-.23481923E-01	0.	.23481923E-01	2.42176	1.00000	3
0.	0.	.10000000E-02	3.28130	1.00000	4

TROUBLES NEAR A POLE.  
 REDUCING THE STEP SIZE MAY HELP.

SUB-BRANCH STARTING AT X = 1.8765 Y = 15.000

CALCULATION STEP SIZE = .1250  
 PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
1.6882589	14.404018	14.502618	.440195	.11641	0
1.5078167	13.805634	13.887730	.443011	.10857	0
1.3350548	13.204990	13.272307	.446258	.10059	0
1.1699938	12.602184	12.656379	.450030	.09244	0
1.0126614	11.997316	12.039978	.454447	.08411	0
.86308430	11.390482	11.423134	.459671	.07556	0
.72128787	10.781784	10.805884	.465914	.06675	0
.58729672	10.171320	10.188261	.473470	.05764	0
.46113464	9.5591901	9.5703062	.482744	.04818	0
.34282477	8.9454942	8.9520610	.494321	.03830	0
.23238997	8.3303324	8.3335733	.509066	.02789	0
.12985330	7.7136050	7.7140979	.528311	.01683	0
.35238873E-01	7.0960122	7.0960997	.554211	.00497	0
0.	6.8512476	6.8512476	.567033	.00000	4
-.83515543E-01	6.2318568	6.2324164	.609024	.01340	0
-.15903229	5.6114400	5.6136931	.672690	.02833	0
-.22650526	4.9900970	4.9952350	.778263	.04534	0
-.28585856	4.3679260	4.3772700	.981052	.06531	0
-.33695534	3.7450227	3.7601508	1.50366	.08961	0
-.37950859	3.1214780	3.1444637	5.27637	.12069	0
-.39156000	2.9159000	2.9420727	0.	.13309	2

41

M = 0.4  
h = 20K ft

## OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 41

## CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

REGION OF CALCULATION-REAL: CC= -15.0 TO AA= 10.0

IMAJ: DD= -.100E-01 TO BB= 15.0

BRANCH STARTING AT (-.012824) + J(.11286)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-.012824) + J(-.11286)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (1.8793) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-2.6203) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >L

BRANCH NUMBER 4

CALCULATION STEP SIZE = .1250

PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
-2.6203000	0.	2.6203000	0.	1.00000	1
-3.2453000	0.	3.2453000	.100248	1.00000	0
-3.8703000	0.	3.8703000	.179688	1.00000	0
-4.4953000	0.	4.4953000	.241082	1.00000	0
-5.1203000	0.	5.1203000	.288335	1.00000	0
-5.7453000	0.	5.7453000	.324922	1.00000	0
-6.3703000	0.	6.3703000	.353551	1.00000	0
-6.9953000	0.	6.9953000	.376229	1.00000	0
-7.6203000	0.	7.6203000	.394423	1.00000	0
-8.2453000	0.	8.2453000	.409198	1.00000	0
-8.8703000	0.	8.8703000	.421338	1.00000	0
-9.4953000	0.	9.4953000	.431419	1.00000	0
-10.120300	0.	10.120300	.439975	1.00000	0
-10.745300	0.	10.745300	.447033	1.00000	0
-11.370300	0.	11.370300	.453144	1.00000	0
-11.995300	0.	11.995300	.458401	1.00000	0
-12.620300	0.	12.620300	.462955	1.00000	0
-13.245300	0.	13.245300	.466926	1.00000	0
-13.870300	0.	13.870300	.470411	1.00000	0
-14.495300	0.	14.495300	.473484	1.00000	0
-15.120300	0.	15.120300	.476210	1.00000	0

BOUNDARY

SUB-BRANCH STARTING AT X = 1.8867 Y = 15.000

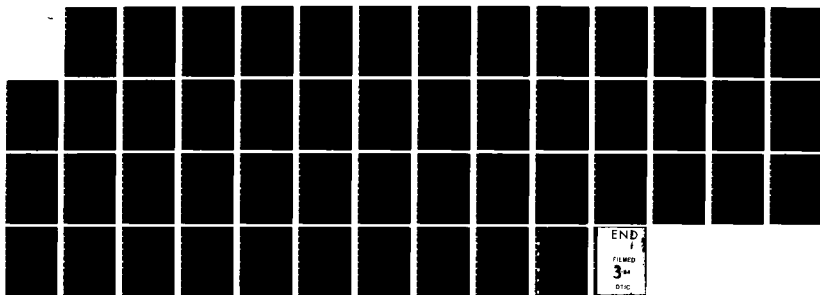
HD-A137 814

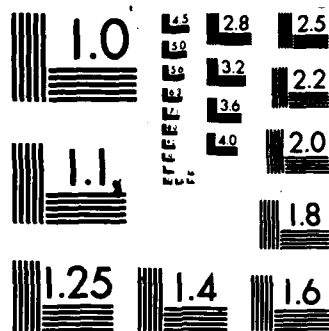
DESIGN OF LONGITUDINAL CONTROL LAWS FOR THE X-29A  
BACKUP ANALOG FLIGHT CO. (U) AIR FORCE INST OF TECH  
WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI... H L EMRICK  
SEP 83 AFIT/GAE/AA/83S-3 F/G 1/3

2/2

UNCLASSIFIED

NL





MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

CALCULATION STEP SIZE = .1250  
 PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
1.7088453	14.400851	14.501885	.542840	.11784	0
1.5384859	13.799521	13.885018	.545764	.11080	0
1.3753989	13.196177	13.267661	.549133	.10367	0
1.2194073	12.590909	12.649840	.553041	.09641	0
1.0711328	11.983805	12.031580	.557614	.08903	0
.92999572	11.374953	11.412907	.563013	.08149	0
.79621494	10.764442	10.793849	.569455	.07377	0
.66980776	10.152363	10.174434	.577233	.06583	0
.55078963	9.5388032	9.5546918	.586756	.05765	0
.43917385	8.9238541	8.9346542	.598605	.04915	0
.33497109	8.3076056	8.3143560	.613634	.04029	0
.23818861	7.6901482	7.6938361	.633142	.03096	0
.14882895	7.0715729	7.0731389	.659204	.02104	0
.66887810E-01	6.4519713	6.4523180	.695323	.01037	0
0.	5.8980823	5.8980823	.741144	.00000	4
-.67958627E-01	5.2767916	5.2772291	.810872	.01288	0
-.12859856	4.6547437	4.6565198	.951268	.02762	0
-.18204014	4.0320360	4.0361433	1.21742	.04510	0
-.22852090	3.4087697	3.4164210	1.97851	.06689	0
-.26856131	2.7850559	2.7979746	14.9320	.09598	0
-.27326000	2.7054000	2.7191653	0.	.10049	2

GAIN=-1,41

M = 0.5  
h = 40K ft

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 41

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

REGION OF CALCULATION-REAL: CC= -15.0 TO AA= 10.0  
IMAJ: DD= -.100E-01 TO BB= 15.0

BRANCH STARTING AT (-.015728) + J(.094651)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-.015728) + J(-.094651)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (1.578) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-1.9912) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >L

BRANCH NUMBER 4

CALCULATION STEP SIZE = .1250  
PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
-1.9912000	0.	1.9912000	0.	1.00000	1
-2.6162000	0.	2.6162000	.192622	1.00000	0
-3.2412000	0.	3.2412000	.337893	1.00000	0
-3.8662000	0.	3.8662000	.443711	1.00000	0
-4.4912000	0.	4.4912000	.520722	1.00000	0
-5.1162000	0.	5.1162000	.577490	1.00000	0
-5.7412000	0.	5.7412000	.620081	1.00000	0
-6.3662000	0.	6.3662000	.652636	1.00000	0
-6.9912000	0.	6.9912000	.677973	1.00000	0
-7.6162000	0.	7.6162000	.698022	1.00000	0
-8.2412000	0.	8.2412000	.714131	1.00000	0
-8.8662000	0.	8.8662000	.727253	1.00000	0
-9.4912000	0.	9.4912000	.738074	1.00000	0
-10.116200	0.	10.116200	.747100	1.00000	0
-10.741200	0.	10.741200	.754704	1.00000	0
-11.366200	0.	11.366200	.761169	1.00000	0
-11.991200	0.	11.991200	.766711	1.00000	0
-12.616200	0.	12.616200	.771498	1.00000	0
-13.241200	0.	13.241200	.775663	1.00000	0
-13.866200	0.	13.866200	.779308	1.00000	0
-14.491200	0.	14.491200	.782517	1.00000	0
-15.116200	0.	15.116200	.785358	1.00000	0

BOUNDARY

SUB-BRANCH STARTING AT X = 1.7188 Y = 15.000

CALCULATION STEP SIZE = .1250  
 PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
1.5644749	14.394344	14.479113	.854319	.10805	0
1.4167627	13.787053	13.859655	.857367	.10222	0
1.2754201	13.178247	13.239822	.860872	.09633	0
1.1404618	12.567995	12.619634	.864931	.09037	0
1.0119017	11.956363	11.999107	.869668	.08433	0
.88975279	11.343418	11.378260	.875245	.07820	0
.77402687	10.729228	10.757112	.881877	.07195	0
.66473463	10.113861	10.135683	.889852	.06558	0
.56188535	9.4973844	9.5139911	.899566	.05906	0
.46548660	8.8798661	8.8920582	.911575	.05235	0
.37554382	8.2613745	8.2699058	.926685	.04541	0
.29205953	7.6419780	7.6475569	.946096	.03819	0
.21503218	7.0217455	7.0250372	.971672	.03061	0
.14445407	6.4007460	6.4023758	1.00645	.02256	0
.00307629E-01	5.7790492	5.7796071	1.05572	.01389	0
.22558543E-01	5.1567255	5.1567748	1.12941	.00437	0
0.	4.8927653	4.8927653	1.17244	.00000	4
-.48769098E-01	4.2696735	4.2699520	1.32336	.01142	0
-.91415851E-01	3.6461325	3.6472783	1.62031	.02506	0
-.12830949	3.0222243	3.0249467	2.41511	.04242	0
-.16033831	2.3980466	2.4034008	9.18036	.06671	0
-.16858000	2.2258000	2.2321749	0.	.07552	2

GAIN=-1  
41

M = 0.6  
h = 0 ft

OPTION >

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 41

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

REGION OF CALCULATION-REAL: CC= -15.0 TO AA= 10.0

IMAJ: DD= -.100E-01 TO BB= 15.0

BRANCH STARTING AT (-.010817) + J(.06691)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-.010817) + J(-.06691)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (4.0251) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-6.6276) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >L

BRANCH NUMBER 4

CALCULATION STEP SIZE = .1250  
PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
-6.6276000	0.	6.6276000	0.	1.00000	1
-7.2526000	0.	7.2526000	.785703E-02	1.00000	0
-7.8776000	0.	7.8776000	.150327E-01	1.00000	0
-8.5026000	0.	8.5026000	.215300E-01	1.00000	0
-9.1276000	0.	9.1276000	.273804E-01	1.00000	0
-9.7526000	0.	9.7526000	.326305E-01	1.00000	0
-10.377600	0.	10.377600	.373339E-01	1.00000	0
-11.002600	0.	11.002600	.415454E-01	1.00000	0
-11.627600	0.	11.627600	.453178E-01	1.00000	0
-12.252600	0.	12.252600	.487003E-01	1.00000	0
-12.877600	0.	12.877600	.517377E-01	1.00000	0
-13.502600	0.	13.502600	.544702E-01	1.00000	0
-14.127600	0.	14.127600	.569332E-01	1.00000	0
-14.752600	0.	14.752600	.591582E-01	1.00000	0
-15.002600	0.	15.002600	.599875E-01	1.00000	0

BOUNDARY

SUB-BRANCH STARTING AT X = -.97656E-02 Y = -.10000E-01

CALCULATION STEP SIZE = .1250  
PRINTING STEP SIZE = .6250

LOCUS REAL LOCUS IMAG DIST TO ORIGIN GAIN ZETA CD  
TROUBLE IN FINDING THE NEXT POINT TO WITHIN 10E-5 ACCURACY.  
REDUCING THE STEP SIZE MAY HELP.



SUB-BRANCH STARTING AT X = .32275

Y = 15.000

CALCULATION STEP SIZE = .1250

PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
.28565840	14.386070	14.387540	.127309	.01429	0
.93724508E-01	13.771177	13.771495	.131615	.00681	0
0.	13.233617	13.233617	.136079	.00000	4
-.10277008	12.617126	12.617545	.142234	.00815	0
-.20063426	11.999837	12.001514	.149864	.01672	0
-.29358946	11.381790	11.385576	.159531	.02579	0
-.38163014	10.763024	10.769787	.172114	.03544	0
-.46475117	10.143577	10.154218	.189069	.04577	0
-.54294786	9.5234897	9.5389543	.213003	.05692	0
-.61621598	8.9028007	8.9241012	.249063	.06905	0
-.68455185	8.2815494	8.3097937	.309037	.08238	0
-.74795239	7.6597750	7.6962059	.427117	.09718	0
-.80641522	7.0375169	7.0835690	.761090	.11384	0
-.85993874	6.4148145	6.4721974	7.37146	.13287	0
-.86537000	6.3482000	6.4069110	0.	.13507	2

GAIN=-1,41

M = 0.6  
h = 20K ft

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 41

CODE

0-LOCUS PT.

1-POLE

2-ZERO

3-BREAK PT.

4-IMAGINARY AXIS

5-SENSITIVITY PT.

REGION OF CALCULATION-REAL: CC= -15.0 TO AA= 10.0  
IMAJ: DD= -.100E-01 TO BB= 15.0

BRANCH STARTING AT (-.0070548) + J(.076164)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-.0070548) + J(-.076164)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (2.8675) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-4.0591) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >L

BRANCH NUMBER 4

CALCULATION STEP SIZE = .1250  
PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
-4.0591000	0.	4.0591000	0.	1.00000	1
-4.6841000	0.	4.6841000	.283670E-01	1.00000	0
-5.3091000	0.	5.3091000	.530369E-01	1.00000	0
-5.9341000	0.	5.9341000	.741261E-01	1.00000	0
-6.5591000	0.	6.5591000	.920019E-01	1.00000	0
-7.1841000	0.	7.1841000	.107109	1.00000	0
-7.8091000	0.	7.8091000	.119883	1.00000	0
-8.4341000	0.	8.4341000	.130713	1.00000	0
-9.0591000	0.	9.0591000	.139931	1.00000	0
-9.6841000	0.	9.6841000	.147815	1.00000	0
-10.309100	0.	10.309100	.154591	1.00000	0
-10.934100	0.	10.934100	.160446	1.00000	0
-11.559100	0.	11.559100	.165532	1.00000	0
-12.184100	0.	12.184100	.169971	1.00000	0
-12.809100	0.	12.809100	.173865	1.00000	0
-13.434100	0.	13.434100	.177297	1.00000	0
-14.059100	0.	14.059100	.180336	1.00000	0
-14.684100	0.	14.684100	.183038	1.00000	0
-15.059100	0.	15.059100	.184518	1.00000	0

BOUNDARY

SUB-BRANCH STARTING AT X = -.14648E-02 Y = -.10000E-01

CALCULATION STEP SIZE = .1250  
PRINTING STEP SIZE = .6250

LOCUS REAL LOCUS IMAG DIST TO ORIGIN GAIN ZETA CD  
TROUBLE IN FINDING THE NEXT POINT TO WITHIN 10E-5 ACCURACY.

REDUCING THE STEP SIZE MAY HELP.

SUB-BRANCH STARTING AT X = .87158

Y = 15.000

CALCULATION STEP SIZE = .1250

PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
.75816272	14.385388	14.485345	.251621	.05263	0
.64982836	13.769843	13.785167	.255088	.04714	0
.54623875	13.153498	13.164827	.259135	.04149	0
.44737558	12.536359	12.544339	.263902	.03566	0
.35326783	11.918486	11.923720	.269578	.02963	0
.26391248	11.299988	11.302989	.276422	.02335	0
.17931365	10.688661	10.682166	.284795	.01679	0
.99474938E-01	10.060783	10.061275	.295216	.00989	0
.24399187E-01	9.4403181	9.4403417	.308460	.00258	0
0.	9.2296162	9.2296162	.313792	.00000	4
-.68689623E-01	8.6084038	8.6086778	.332824	.00798	0
-.13261883	7.9866833	7.9877843	.358818	.01660	0
-.19178387	7.3644915	7.3669883	.396823	.02683	0
-.24618968	6.7418655	6.7463598	.453818	.03649	0
-.29584786	6.1188428	6.1259988	.549722	.04829	0
-.34077238	5.4954618	5.5060164	.745527	.06189	0
-.38099543	4.8717588	4.8866332	1.33179	.07797	0
-.41656983	4.2477726	4.2681498	48.1548	.09760	0
-.41742088	4.2318888	4.2523371	0.	.09816	2

GAIN=-1,41

M = 0.6  
h = 40K ft

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 41

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT..

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

REGION OF CALCULATION-REAL: CC= -15.0 TO AA= 10.0  
IMAJ: DD= -.100E-01 TO BB= 15.0

BRANCH STARTING AT (-.010247) + J(.000888)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-.010247) + J(-.000888)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (1.9091) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-2.4136) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >L

BRANCH NUMBER 4

CALCULATION STEP SIZE = .1250  
PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
-2.4136000	0.	2.4136000	0.	1.00000	1
-3.0386000	0.	3.0386000	.112034	1.00000	0
-3.6636000	0.	3.6636000	.201191	1.00000	0
-4.2886000	0.	4.2886000	.270014	1.00000	0
-4.9136000	0.	4.9136000	.322790	1.00000	0
-5.5386000	0.	5.5386000	.363459	1.00000	0
-6.1636000	0.	6.1636000	.395121	1.00000	0
-6.7886000	0.	6.7886000	.420076	1.00000	0
-7.4136000	0.	7.4136000	.439997	1.00000	0
-8.0386000	0.	8.0386000	.456101	1.00000	0
-8.6636000	0.	8.6636000	.469274	1.00000	0
-9.2886000	0.	9.2886000	.480168	1.00000	0
-9.9136000	0.	9.9136000	.489270	1.00000	0
-10.538600	0.	10.538600	.496947	1.00000	0
-11.163600	0.	11.163600	.503477	1.00000	0
-11.788600	0.	11.788600	.509076	1.00000	0
-12.413600	0.	12.413600	.513911	1.00000	0
-13.038600	0.	13.038600	.518115	1.00000	0
-13.663600	0.	13.663600	.521792	1.00000	0
-14.288600	0.	14.288600	.525028	1.00000	0
-14.913600	0.	14.913600	.527889	1.00000	0
-15.038600	0.	15.038600	.528421	1.00000	0

BOUNDARY

SUB-BRANCH STARTING AT X = 1.2607 Y = 15.000

CALCULATION STEP SIZE = .1250  
PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
1.1398766	14.386888	14.431886	.597081	.07898	0
1.0241511	13.772609	13.810636	.600275	.07416	0
.91347338	13.157489	13.189160	.603958	.06926	0
.80785028	12.541488	12.567472	.608237	.06428	0
.70726805	11.924625	11.945582	.613252	.05921	0
.61179240	11.306965	11.323505	.619185	.05403	0
.52136835	10.688543	10.701251	.626278	.04872	0
.43602014	10.069399	10.078835	.634864	.04326	0
.35575101	9.4495771	9.4562712	.645404	.03762	0
.28056295	8.8291179	8.8335745	.658562	.03176	0
.21045628	8.2080639	8.2107616	.675317	.02563	0
.14542894	7.5864577	7.5878514	.697170	.01917	0
.85475344E-01	6.9643415	6.9648660	.726542	.01227	0
.30584498E-01	6.3417582	6.3418320	.767567	.00482	0
0.	5.9670882	5.9670882	.800803	.00000	4
-.46830595E-01	5.3438468	5.3440520	.879447	.00876	0
-.88687694E-01	4.7202515	4.7210846	1.01165	.01879	0
-.12566535	4.0963478	4.0982749	1.27115	.03066	0
-.15795046	3.4721836	3.4757743	1.97110	.04544	0
-.18595203	2.8478122	2.8538767	8.66802	.06516	0
-.19211000	2.6983000	2.7051302	0.	.07102	2

GAIN=-1.41

M = 0.8  
h = 0 ft

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 41

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

REGION OF CALCULATION-REAL: CC= -10.0

IMAJ: DD= -12.0

TO AA= 6.00

TO BB= 12.0

BRANCH STARTING AT (-.016487) + J(.041562)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-.016487) + J(-.041562)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (5.4774) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-9.3031) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >L

BRANCH NUMBER 4

CALCULATION STEP SIZE = .1280  
PRINTING STEP SIZE = .6400

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
-9.3031000	0.	9.3031000	0.	1.00000	1
-9.9431000	0.	9.9431000	.299934E-02	1.00000	0
-10.071100	0.	10.071100	.357729E-02	1.00000	0

BOUNDARY

SUB-BRANCH STARTING AT X = -.93900 Y = -12.000

CALCULATION STEP SIZE = .1280  
PRINTING STEP SIZE = .6400

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
-1.0123030	-11.364213	11.409211	.172771	.08873	0
-1.0814220	-10.727958	10.782326	.222210	.10030	0
-1.1465542	-10.091282	10.156208	.325321	.11289	0
-1.2076961	-9.4542099	9.5310343	.669008	.12671	0
-1.2572000	-8.9047000	8.9930103	0.	.13980	2

SUB-BRANCH STARTING AT X = -.93900 Y = 12.000

CALCULATION STEP SIZE = .1280  
PRINTING STEP SIZE = .6400

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
-1.0123030	11.364213	11.409211	.172771	.08873	0

-1.0814220	10.727958	10.782326	.222210	.10030	0
-1.1465542	10.091282	10.156208	.325321	.11289	0
-1.2076961	9.4542099	9.5310343	.669008	.12671	0
-1.2572000	8.9047000	8.9930103	0.	.13980	2

GAIN=-1,41

M = 1.0  
h = 20K ft

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 41

CODE

0-LOCUS PT.

1-POLE

2-ZERO

3-BREAK PT.

4-IMAGINARY AXIS

5-SENSITIVITY PT.

REGION OF CALCULATION-REAL: CC= -15.0 TO AA= 10.0

IMAJ: DD= -.100E-01 TO BB= 15.0

BRANCH STARTING AT (-.03975) + J(0.)

TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (.016974) + J(0.)

TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (3.9644) + J(0.)

TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-6.0004) + J(0.)

TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >L

BRANCH NUMBER 4

CALCULATION STEP SIZE = .1250

PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
-6.0004000	0.	6.0004000	0.	1.00000	1
-6.6254000	0.	6.6254000	.630787E-02	1.00000	0
-7.2504000	0.	7.2504000	.120895E-01	1.00000	0
-7.8754000	0.	7.8754000	.173338E-01	1.00000	0
-8.5004000	0.	8.5004000	.220579E-01	1.00000	0
-9.1254000	0.	9.1254000	.262952E-01	1.00000	0
-9.7504000	0.	9.7504000	.300871E-01	1.00000	0
-10.375400	0.	10.375400	.334773E-01	1.00000	0
-11.000400	0.	11.000400	.365085E-01	1.00000	0
-11.625400	0.	11.625400	.392212E-01	1.00000	0
-12.250400	0.	12.250400	.416522E-01	1.00000	0
-12.875400	0.	12.875400	.438345E-01	1.00000	0
-13.500400	0.	13.500400	.457977E-01	1.00000	0
-14.125400	0.	14.125400	.475675E-01	1.00000	0
-14.750400	0.	14.750400	.491667E-01	1.00000	0
-15.000400	0.	15.000400	.497631E-01	1.00000	0

BOUNDARY

SUB-BRANCH STARTING AT X = .65527

Y = 15.000

CALCULATION STEP SIZE = .1250

PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
.53325417	14.387029	14.396908	.989728E-01	.03704	0
.41642920	13.773046	13.779340	.102104	.03022	0
.30470242	13.158115	13.161643	.105860	.02315	0
.19809153	12.542279	12.543843	.110433	.01579	0



.98573841E-01	11.925578	11.925969	.116099	.00810	0
.18631845E-03	11.308057	11.308057	.123272	.00002	0
0.	11.306830	11.306830	.123288	.00000	4
-.91250569E-01	10.688529	10.688919	.132620	.00854	0
-.17736810	10.069492	10.071054	.145179	.01761	0
-.25834673	9.4497622	9.4532930	.162873	.02733	0
-.33418096	8.8293817	8.8357036	.189453	.03782	0
-.40486569	8.2083933	8.2183719	.233450	.04926	0
-.47039622	7.5868400	7.6014086	.319288	.06188	0
-.53076832	6.9647644	6.9849594	.556509	.07599	0
-.58597825	6.3422094	6.3692221	4.00809	.09200	0
-.59373000	6.2498000	6.2779388	0.	.09457	2

GAIN=-1,41

M = 1.0  
h = 40K ft

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 41

CODE  
0-LOCUS PT.                      2-ZERO                      4-IMAGINARY AXIS  
1-POLE                            3-BREAK PT.                      5-SENSITIVITY PT.

REGION OF CALCULATION-REAL: CC= -15.0                      TO AA= 10.0

IMAJ: DD= -.100E-01                      TO BB= 15.0

BRANCH STARTING AT (-.0067527) + J(.047972)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-.0067527) + J(-.047972)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (2.6941) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-3.5394) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >L

BRANCH NUMBER 4

CALCULATION STEP SIZE = .1250  
PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
-3.5394000	0.	3.5394000	0.	1.00000	1
-4.1644000	0.	4.1644000	.258865E-01	1.00000	0
-4.7894000	0.	4.7894000	.485596E-01	1.00000	0
-5.4144000	0.	5.4144000	.679631E-01	1.00000	0
-6.0394000	0.	6.0394000	.843709E-01	1.00000	0
-6.6644000	0.	6.6644000	.981779E-01	1.00000	0
-7.2894000	0.	7.2894000	.109791	1.00000	0
-7.9144000	0.	7.9144000	.119580	1.00000	0
-8.5394000	0.	8.5394000	.127865	1.00000	0
-9.1644000	0.	9.1644000	.134910	1.00000	0
-9.7894000	0.	9.7894000	.140933	1.00000	0
-10.414400	0.	10.414400	.146110	1.00000	0
-11.039400	0.	11.039400	.150585	1.00000	0
-11.664400	0.	11.664400	.154473	1.00000	0
-12.289400	0.	12.289400	.157870	1.00000	0
-12.914400	0.	12.914400	.160852	1.00000	0
-13.539400	0.	13.539400	.163482	1.00000	0
-14.164400	0.	14.164400	.165812	1.00000	0
-14.789400	0.	14.789400	.167886	1.00000	0
-15.039400	0.	15.039400	.168652	1.00000	0

BOUNDARY

SUB-BRANCH STARTING AT X = -.34180E-02                      Y = -.10000E-01

CALCULATION STEP SIZE = .1250  
PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
------------	------------	----------------	------	------	----

SUB-BRANCH STARTING AT X = .97900

Y = 15.000

CALCULATION STEP SIZE = .1250

PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
.86964982	14.384644	14.410908	.221735	.06035	0
.76523054	13.768438	13.789679	.224482	.05549	0
.66538350	13.151458	13.168280	.227685	.05053	0
.57011393	12.533763	12.546723	.231454	.04544	0
.47942675	11.915379	11.925020	.235935	.04020	0
.39332653	11.296340	11.303185	.241330	.03480	0
.31181746	10.676679	10.681231	.247916	.02919	0
.23490332	10.056431	10.059174	.256094	.02335	0
.16258738	9.4356299	9.4370306	.266456	.01723	0
.94872308E-01	8.8143104	8.8148209	.279914	.01076	0
.31760032E-01	8.1925064	8.1925680	.297953	.00388	0
0.	7.8609146	7.8609146	.310255	.00000	4
-.56051611E-01	7.2384344	7.2386515	.341009	.00774	0
-.10750463	6.6155573	6.6164308	.388255	.01625	0
-.15435995	5.9923175	5.9943053	.468770	.02575	0
-.19662648	5.3687497	5.3723491	.633019	.03660	0
-.23432109	4.7448888	4.7506711	1.13399	.04932	0
-.26742000	4.1219000	4.1305657	0.	.06474	2

GAIN=-1.41

M = 1.2  
h = 20K ft

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 4:

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

REGION OF CALCULATION-REAL: CC= -10.0  
IMAJ: DD= -12.0

TO AA= 6.00  
TO BB= 12.0

BRANCH STARTING AT (-.091117) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (.053264) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (3.4147) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-5.7362) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >L

BRANCH NUMBER 4

CALCULATION STEP SIZE = .1200  
PRINTING STEP SIZE = .6400

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
-5.7362000	0.	5.7362000	0.	1.00000	1
-6.3762000	0.	6.3762000	.401956E-02	1.00000	0
-7.0162000	0.	7.0162000	.775702E-02	1.00000	0
-7.6562000	0.	7.6562000	.111841E-01	1.00000	0
-8.2962000	0.	8.2962000	.142970E-01	1.00000	0
-8.9362000	0.	8.9362000	.171069E-01	1.00000	0
-9.5762000	0.	9.5762000	.196337E-01	1.00000	0
-10.088200	0.	10.088200	.214674E-01	1.00000	0
BOUNDARY					

SUB-BRANCH STARTING AT X = .35800 Y = -12.000

CALCULATION STEP SIZE = .1200  
PRINTING STEP SIZE = .6400

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
.22127242	-11.374780	11.376932	.818842E-01	.01945	0
.92018295E-01	-10.747972	10.748365	.862560E-01	.00856	0
0.	-10.277548	10.277548	.944845E-01	.00000	4
-.11643212	-9.6482320	9.6489345	.105814	.01207	0
-.22552421	-9.0176018	9.0204215	.122943	.02500	0
-.32725949	-8.3857431	8.3921264	.151585	.03900	0
-.42163005	-7.7527425	7.7641991	.208528	.05430	0
-.50863180	-7.1186871	7.1368349	.373609	.07127	0
-.58826625	-6.4836643	6.5102964	5.59816	.09036	0
-.59323000	-6.4420000	6.4692570	0.	.09170	2

SUB-BRANCH STARTING AT X = .35800

Y = 12.000

CALCULATION STEP SIZE = .1280  
PRINTING STEP SIZE = .6400

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
.22127242	11.374780	11.376932	.818842E-01	.01945	0
.92818295E-01	10.747972	10.748365	.002560E-01	.00056	0
0.	10.277548	10.277548	.944845E-01	.00000	0
-.11643212	9.6482320	9.6489345	.105814	.01207	0
-.22552421	9.0176018	9.0204215	.122943	.02500	0
-.32725949	8.3857431	8.3921264	.151585	.03900	0
-.42163005	7.7527425	7.7641991	.208528	.05430	0
-.50863180	7.1186871	7.1368349	.373609	.07127	0
-.58826625	6.4836643	6.5102964	5.59816	.09030	0
-.59323000	6.4420000	6.4692570	0.	.09170	2

GAIN=-1,41

M = 1.4  
h = 40K ft

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 41

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

REGION OF CALCULATION-REAL: CC= -15.0 TO AA= 10.0  
IMAJ: DD= -.100E-01 TO BB= 15.0

BRANCH STARTING AT (-.0082773) + J(.039491)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-.0082773) + J(-.039491)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (1.5136) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >S

BRANCH STARTING AT (-2.596) + J(0.)  
TYPE L TO LIST, S TO SKIP, OR \* TO ABORT >L

BRANCH NUMBER 4

CALCULATION STEP SIZE = .1250  
PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA-	CD
-2.5960000	0.	2.5960000	0.	1.00000	1
-3.2210000	0.	3.2210000	.927762E-02	1.00000	0
-3.8460000	0.	3.8460000	.185849E-01	1.00000	0
-4.4710000	0.	4.4710000	.273683E-01	1.00000	0
-5.0960000	0.	5.0960000	.353435E-01	1.00000	0
-5.7210000	0.	5.7210000	.424133E-01	1.00000	0
-6.3460000	0.	6.3460000	.485914E-01	1.00000	0
-6.9710000	0.	6.9710000	.539481E-01	1.00000	0
-7.5960000	0.	7.5960000	.585765E-01	1.00000	0
-8.2210000	0.	8.2210000	.625732E-01	1.00000	0
-8.8460000	0.	8.8460000	.660288E-01	1.00000	0
-9.4710000	0.	9.4710000	.690241E-01	1.00000	0
-10.096000	0.	10.096000	.716289E-01	1.00000	0
-10.721000	0.	10.721000	.739025E-01	1.00000	0
-11.346000	0.	11.346000	.758950E-01	1.00000	0
-11.971000	0.	11.971000	.776482E-01	1.00000	0
-12.596000	0.	12.596000	.791971E-01	1.00000	0
-13.221000	0.	13.221000	.805711E-01	1.00000	0
-13.846000	0.	13.846000	.817945E-01	1.00000	0
-14.471000	0.	14.471000	.828881E-01	1.00000	0
-15.096000	0.	15.096000	.838691E-01	1.00000	0
BOUNDARY					

SUB-BRANCH STARTING AT X = -.78125E-02 Y = -.10000E-01

CALCULATION STEP SIZE = .1250  
PRINTING STEP SIZE = .6250

TROUBLE IN FINDING THE NEXT POINT TO WITHIN 10E-5 ACCURACY.  
REDUCING THE STEP SIZE MAY HELP.

SUB-BRANCH STARTING AT X = 1.6631 Y = 15.000

CALCULATION STEP SIZE = .1250  
PRINTING STEP SIZE = .6250

LOCUS REAL	LOCUS IMAG	DIST TO ORIGIN	GAIN	ZETA	CD
1.4897629	14.399518	14.476378	.111569	.10291	0
1.3236763	13.796993	13.860344	.112999	.09550	0
1.1646991	13.192554	13.243867	.114674	.08794	0
1.0128532	12.586284	12.626972	.116656	.08021	0
.86815994	11.978267	12.009687	.119029	.07229	0
.73063920	11.368588	11.392042	.121989	.06414	0
.60031010	10.757331	10.774068	.125460	.05572	0
.47719073	10.144581	10.155798	.129925	.04699	0
.36129814	9.5304234	9.5372694	.135673	.03788	0
.25264841	8.9149432	8.9185225	.143297	.02833	0
.15125657	8.2982258	8.2996042	.153809	.01822	0
.57136678E-01	7.6803568	7.6805693	.169082	.00744	0
0.	7.2791211	7.2791211	.183212	.00000	4
-.82107510E-01	6.6595414	6.6600475	.217200	.01233	0
-.15690786	6.0390371	6.0410752	.265023	.02597	0
-.22439375	5.4176948	5.4223398	.481565	.04138	0
-.28455585	4.7956006	4.8040355	6.02775	.05923	0
-.28872000	4.7495000	4.7582675	0.	.06068	2

42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL):>-1.2,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.3  
h = 0 ft

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -1.200

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = -2.923

+OR- KTOL = .2436

ROOTS OF INTEREST

X = .43183E-02	Y = -.60757E-01
X = .43183E-02	Y = .60757E-01
X = -.29209	Y = -4.0147
X = -.29209	Y = 4.0147

REGION OF CALCULATION-REAL: CC= -5.00

TO AA= 1.00

IMAJ: DD= -3.00

TO BB= 3.00

BRANCH STARTING AT (-.014311) + J(.14132)

42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL):>-1.2,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.4  
h = 20K ft

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -1.200

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = -2.344

+OR- KTOL = .1954

ROOTS OF INTEREST

X = .46977E-02	Y = -.52395E-01
X = .46977E-02	Y = .52395E-01
X = -.17979	Y = 4.0600
X = -.17979	Y = -4.0600

REGION OF CALCULATION-REAL: CC= -5.00

TO AA= 1.00

IMAJ: DD= -3.00

TO BB= 3.00

BRANCH STARTING AT (-.012824) + J(.11286)

42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL):>-1.2,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.5  
h = 40K ft

CODE

0-LOCUS PT.

2-ZERO

4-IMAGINARY AXIS



GAIN OF INTEREST (GAIN) = -1.200 +OR- GTOL = .1000  
 SENSITIVITY OF INTEREST (OLK) = -1.460 +OR- KTOL = .1217

ROOTS OF INTEREST  
 X = .54698E-02 Y = -.51752E-01  
 X = .54698E-02 Y = .51752E-01  
 X = -.11778E-01 Y = -4.7494  
 X = -.11778E-01 Y = 4.7494

REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00  
 IMAJ: DD= -3.00 TO BB= 3.00  
 BRANCH STARTING AT (-.015728) + J(.094651)  
 42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -1.2  
 ,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42 M = 0.6  
 h = 0 ft

CODE  
 0-LOCUS PT. 2-ZERO 4-IMAGINARY AXIS  
 1-POLE 3-BREAK PT. 5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -1.200 +OR- GTOL = .1000  
 SENSITIVITY OF INTEREST (OLK) = -13.27 +OR- KTOL = 1.106

ROOTS OF INTEREST  
 X = -.98176E-02 Y = -.91793E-02  
 X = -.98176E-02 Y = .91793E-02  
 X = -.82973 Y = 6.7733  
 X = -.82973 Y = -6.7733

REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00  
 IMAJ: DD= -3.00 TO BB= 3.00  
 BRANCH STARTING AT (-.010817) + J(.06691)  
 42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -1.2, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42 M = 0.6  
 h = 20K ft

CODE  
 0-LOCUS PT. 2-ZERO 4-IMAGINARY AXIS  
 1-POLE 3-BREAK PT. 5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -1.200 +OR- GTOL = .1000  
 SENSITIVITY OF INTEREST (OLK) = -5.409 +OR- KTOL = .4507

ROOTS OF INTEREST  
 X = -.19634E-02 Y = -.24173E-01  
 X = -.19634E-02 Y = .24173E-01  
 X = -.37512 Y = -4.9676  
 X = -.37512 Y = 4.9676

IMAJ: DD= -3.00 TO BB= 3.00  
 BRANCH STARTING AT (-.0070548) + J(.076164)  
 42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -1.2, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.6  
 h = 40K

CODE  
 0-LOCUS PT. 2-ZERO 4-IMAGINARY AXIS  
 1-POLE 3-BREAK PT. 5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -1.200 +OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = -2.129 +OR- KTOL = .1774

ROOTS OF INTEREST

X = .40106E-02	Y = -.38340E-01
X = .40106E-02	Y = .38340E-01
X = -.11849	Y = 4.2240
X = -.11849	Y = -4.2240

REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00

IMAJ: DD= -3.00 TO BB= 3.00

BRANCH STARTING AT (-.010247) + J(.080888)  
 42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -1.2, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.8  
 h = 0 ft

CODE  
 0-LOCUS PT. 2-ZERO 4-IMAGINARY AXIS  
 1-POLE 3-BREAK PT. 5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -1.200 +OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = -25.52 +OR- KTOL = 2.127

ROOTS OF INTEREST

X = -.33280E-01	Y = 0.
X = -.64486E-03	Y = 0.
X = -1.2305	Y = 9.2053
X = -1.2305	Y = -9.2053

REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00

IMAJ: DD= -3.00 TO BB= 3.00

BRANCH STARTING AT (-.016487) + J(.041562)  
 42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -1.2, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 1.0  
 h = 20K ft

CODE  
 0-LOCUS PT. 2-ZERO 4-IMAGINARY AXIS  
 1-POLE 3-BREAK PT. 5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -1.200 +OR- GTOL = .1000  
 SENSITIVITY OF INTEREST (OLK) = -16.69 +OR- KTOL = 1.391

ROOTS OF INTEREST  
 X = -.24510E-01 Y = 0.  
 X = .18225E-02 Y = 0.  
 X = -.56668 Y = -6.5666  
 X = -.56668 Y = 6.5666

REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00  
 IMAJ: DD= -3.00 TO BB= 3.00  
 BRANCH STARTING AT (-.03975) + J(0.)  
 42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL):>-1.2,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42 M = 1.0  
 h = 40K ft

CODE  
 0-LOCUS PT. 2-ZERO 4-IMAGINARY AXIS  
 1-POLE 3-BREAK PT. 5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -1.200 +OR- GTOL = .1000  
 SENSITIVITY OF INTEREST (OLK) = -6.167 +OR- KTOL = .5139

ROOTS OF INTEREST  
 X = -.37698E-02 Y = -.12623E-01  
 X = -.37698E-02 Y = .12623E-01  
 X = -.23649 Y = -4.7065  
 X = -.23649 Y = 4.7065

REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00  
 IMAJ: DD= -3.00 TO BB= 3.00  
 BRANCH STARTING AT (-.0067527) + J(.047972)  
 42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL):>-1.2,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42 M = 1.2  
 h = 20K ft

CODE  
 0-LOCUS PT. 2-ZERO 4-IMAGINARY AXIS  
 1-POLE 3-BREAK PT. 5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -1.200 +OR- GTOL = .1000  
 SENSITIVITY OF INTEREST (OLK) = -24.93 +OR- KTOL = 2.078

ROOTS OF INTEREST  
 X = -.34739E-01 Y = 0.  
 X = .32160E-02 Y = 0.  
 X = -.56932 Y = 6.6404  
 X = -.56932 Y = -6.6404

REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00  
 IMAJ: DD= -3.00 TO BB= 3.00

42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -1.2, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 1.4  
h = 40K ft

CODE

0-LOCUS PT.

1-POLE

2-ZERO

3-BREAK PT.

4-IMAGINARY AXIS

5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -1.200

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = -12.28

+OR- KTOL = 1.024

ROOTS OF INTEREST

X = -.15595E-01

Y = 0.

X = -.36254E-03

Y = 0.

X = -.26632

Y = 4.9925

X = -.26632

Y = -4.9925

REGION OF CALCULATION-REAL: CC= -5.00

TO AA= 1.00

IMAJ: DD= -3.00

TO BB= 3.00

BRANCH STARTING AT (-.0082773) + J(.039491)

GAIN=-1,43

ENTER ZETA,RAD,GTOL >.7,1,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 43

M = 0.3  
h = 0 ft

CODE		
0-LOCUS PT.	2-ZERO	4-IMAGINARY AXIS
1-POLE	3-BREAK PT.	5-SENSITIVITY PT.

DAMPING FACTOR OF INTEREST (ZETA) = .7000

GAIN OF INTEREST (GAIN) = 3.652 +OR- GTOL = .3652

SENSITIVITY OF INTEREST (OLK) = -5.713 +OR- KTOL = .5713

ROOTS OF INTEREST

X = .26315E-02	Y = .54473E-01
X = .26315E-02	Y = -.54473E-01
X = -3.1471	Y = 3.2105
X = -3.1471	Y = -3.2105

REGION OF CALCULATION-REAL: CC= -10.0

TO AA= 1.00

IMAJ: DD= -10.0

TO BB= 10.0

BRANCH STARTING AT (.0043183) + J(.060757)

GAIN=-1,43

ENTER ZETA,RAD,GTOL >.7,1,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 43

M = 0.4  
h = 20K ft

CODE		
0-LOCUS PT.	2-ZERO	4-IMAGINARY AXIS
1-POLE	3-BREAK PT.	5-SENSITIVITY PT.

DAMPING FACTOR OF INTEREST (ZETA) = .7000

GAIN OF INTEREST (GAIN) = 3.179 +OR- GTOL = .3179

SENSITIVITY OF INTEREST (OLK) = -5.799 +OR- KTOL = .5799

ROOTS OF INTEREST

X = .37230E-02	Y = .48477E-01
X = .37230E-02	Y = -.48477E-01
X = -3.0781	Y = -3.1401
X = -3.0781	Y = 3.1401

REGION OF CALCULATION-REAL: CC= -10.0

TO AA= 1.00

IMAJ: DD= -10.0

TO BB= 10.0

BRANCH STARTING AT (.0046977) + J(.052395)

GAIN=-1,43

ENTER ZETA,RAD,GTOL >.7,1,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 43

CODE			
0-LOCUS PT.	2-ZERO	4-IMAGINARY AXIS	
1-POLE	3-BREAK PT.	5-SENSITIVITY PT.	

DAMPING FACTOR OF INTEREST (ZETA) = .7000

GAIN OF INTEREST (GAIN) = 2.079 +OR- GTOL = .2079

SENSITIVITY OF INTEREST (OLK) = -6.925 +OR- KTOL = .6925

ROOTS OF INTEREST

X = .51096E-02	Y = .49541E-01	M = 0.5
X = .51096E-02	Y = -.49541E-01	h = 40K ft
X = -3.4739	Y = -3.5441	
X = -3.4739	Y = 3.5441	

REGION OF CALCULATION-REAL: CC= -10.0	TO AA= 1.00
IMAJ: DD= -10.0	TO BB= 10.0

BRANCH STARTING AT (.0054693) + J(.051752)  
GAIN=-1,43

ENTER ZETA,RAD,GTOL >.7,1,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 43

M = 0.6  
h = 0 ft

CODE			
0-LOCUS PT.	2-ZERO	4-IMAGINARY AXIS	
1-POLE	3-BREAK PT.	5-SENSITIVITY PT.	

DAMPING FACTOR OF INTEREST (ZETA) = .7000

GAIN OF INTEREST (GAIN) = 8.545 +OR- GTOL = .8545

SENSITIVITY OF INTEREST (OLK) = -9.343 +OR- KTOL = .9343

ROOTS OF INTEREST

X = -.10139E-01	Y = .57812E-02
X = -.10139E-01	Y = -.57812E-02
X = -5.5008	Y = -5.6117
X = -5.5008	Y = 5.6117

REGION OF CALCULATION-REAL: CC= -10.0	TO AA= 1.00
IMAJ: DD= -10.0	TO BB= 10.0

BRANCH STARTING AT (-.0098176) + J(.0091793)  
GAIN=-1,43

ENTER ZETA,RAD,GTOL >.7,1,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 43

M = 0.6  
h = 20K ft

CODE			
0-LOCUS PT.	2-ZERO	4-IMAGINARY AXIS	
1-POLE	3-BREAK PT.	5-SENSITIVITY PT.	

DAMPING FACTOR OF INTEREST (ZETA) = .7000

GAIN OF INTEREST (GAIN) = 5.254 +OR- GTOL = .5254

SENSITIVITY OF INTEREST (OLK) = -6.918 +OR- KTOL = .6918

ROOTS OF INTEREST

X = -.24673E-02	Y = .21923E-01
X = -.24673E-02	Y = -.21923E-01
X = -3.8336	Y = 3.9110
X = -3.8336	Y = -3.9110

REGION OF CALCULATION-REAL: CC= -10.0 TO AA= 1.00  
 IMAJ: DD= -10.0 TO BB= 10.0  
 BRANCH STARTING AT (-.0019634) + J(.024173)  
 GAIN=-1.43

ENTER ZETA,RAD,GTOL >.7,1,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 43

M = 0.6  
 h = 40K ft

CODE

0-LOCUS PT.	2-ZERO	4-IMAGINARY AXIS
1-POLE	3-BREAK PT.	5-SENSITIVITY PT.

DAMPING FACTOR OF INTEREST (ZETA) = .7000

GAIN OF INTEREST (GAIN) = 3.003 +OR- GTOL = .3003

SENSITIVITY OF INTEREST (OLK) = -6.011 +OR- KTOL = .6011

ROOTS OF INTEREST

X = .35796E-02	Y = .36331E-01
X = .35796E-02	Y = -.36331E-01
X = -3.1234	Y = 3.1865
X = -3.1234	Y = -3.1865

REGION OF CALCULATION-REAL: CC= -10.0 TO AA= 1.00  
 IMAJ: DD= -10.0 TO BB= 10.0  
 BRANCH STARTING AT (.0040106) + J(.03834)  
 GAIN=-1.43

ENTER ZETA,RAD,GTOL >.7,1,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 43

M = 0.8  
 h = 0 ft

CODE

0-LOCUS PT.	2-ZERO	4-IMAGINARY AXIS
1-POLE	3-BREAK PT.	5-SENSITIVITY PT.

DAMPING FACTOR OF INTEREST (ZETA) = .7000

GAIN OF INTEREST (GAIN) = 12.21 +OR- GTOL = 1.221

SENSITIVITY OF INTEREST (OLK) = -12.69 +OR- KTOL = 1.269

ROOTS OF INTEREST

X = -.33683E-01	Y = 0.
X = -.46948E-03	Y = 0.
X = -7.5736	Y = 7.7264
X = -7.5736	Y = -7.7264

REGION OF CALCULATION-REAL: CC= -15.0 TO AA= 1.00  
 IMAJ: DD= -15.0 TO BB= 15.0

GAIN=-1,43

ENTER ZETA,RAD,GTOL >.7,1,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 43

M = 1.0

h = 20K ft

CODE

0-LOCUS PT.

1-POLE

2-ZERO

3-BREAK PT.

4-IMAGINARY AXIS

5-SENSITIVITY PT.

DAMPING FACTOR OF INTEREST (ZETA) = .7000

GAIN OF INTEREST (GAIN) = 10.08

+OR- GTOL = 1.008

SENSITIVITY OF INTEREST (OLK) = -9.190

+OR- KTOL = .9190

ROOTS OF INTEREST

X = -.24328E-01

Y = 0.

X = .14671E-02

Y = 0.

X = -5.1616

Y = -5.2656

X = -5.1616

Y = 5.2656

REGION OF CALCULATION-REAL: CC= -10.0

TO AA= 1.00

IMAJ: DD= -10.0

TO BB= 10.0

BRANCH STARTING AT (-.02451) + J(0.)

GAIN=-1,43.

ENTER ZETA,RAD,GTOL >.7,1,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 43

M = 1.2

h = 20K ft

CODE

0-LOCUS PT.

1-POLE

2-ZERO

3-BREAK PT.

4-IMAGINARY AXIS

5-SENSITIVITY PT.

DAMPING FACTOR OF INTEREST (ZETA) = .7000

GAIN OF INTEREST (GAIN) = 13.45

+OR- GTOL = 1.345

SENSITIVITY OF INTEREST (OLK) = -9.438

+OR- KTOL = .9438

ROOTS OF INTEREST

X = -.33971E-01

Y = 0.

X = .25598E-02

Y = 0.

X = -5.2882

Y = 5.3946

X = -5.2882

Y = -5.3946

REGION OF CALCULATION-REAL: CC= -15.0

TO AA= 1.00

IMAJ: DD= -15.0

TO BB= 15.0

BRANCH STARTING AT (-.034739) + J(0.)



GAIN=-1,43

ENTER ZETA,RAD,GTOL >.7,1,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 43

M = 1.4

h = 40K ft

CODE

0-LOCUS PT.

1-POLE

2-ZERO

3-BREAK PT.

4-IMAGINARY AXIS

5-SENSITIVITY PT.

DAMPING FACTOR OF INTEREST (ZETA) = .7000

GAIN OF INTEREST (GAIN) = 10.25

+OR- GTOL = 1.025

SENSITIVITY OF INTEREST (OLK) = -7.136

+OR- KTOL = .7136

ROOTS OF INTEREST

X = -.29750E-03

Y = 0.

X = -.15835E-01

Y = 0.

X = -3.8340

Y = 3.9113

X = -3.8340

Y = -3.9113

REGION OF CALCULATION-REAL: CC= -10.0

TO AA= 1.00

IMAJ: DD= -10.0

TO BB= 10.0

BRANCH STARTING AT (-.015595) + J(0.)

# Method I

42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL):>3.61,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.3  
h = 0 ft

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = 3.610

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = -5.647

+OR- KTOL = .1564

ROOTS OF INTEREST

X = .26482E-02  
X = .26482E-02  
X = -3.1139  
X = -3.1139

Y = .54535E-01  
Y = -.54535E-01  
Y = -3.2356  
Y = 3.2356

REGION OF CALCULATION-REAL: CC= -5.00

TO AA= 1.00

IMAJ: DD= -3.00

TO BB= 3.00

BRANCH STARTING AT (.0043183) + J(.060757)

42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL):>-3.33,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.4  
h = 20K ft

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -3.330

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 6.074

+OR- KTOL = .1624

ROOTS OF INTEREST

X = .36814E-02  
X = .36814E-02  
X = -3.2155  
X = -3.2155

Y = .48313E-01  
Y = -.48313E-01  
Y = -3.0213  
Y = 3.0213

REGION OF CALCULATION-REAL: CC= -5.00

TO AA= 1.00

IMAJ: DD= -3.00

TO BB= 3.00

BRANCH STARTING AT (.0046977) + J(.052395)

42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL):>-2.87,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.5  
h = 40K ft

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -2.878 +OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 9.561 +OR- KTOL = .3332

ROOTS OF INTEREST

X = .49740E-02	Y = .48774E-01
X = .49740E-02	Y = -.48774E-01
X = -4.7920	Y = 1.5657
X = -4.7920	Y = -1.5657

REGION OF CALCULATION-REAL:	CC= -5.00	TO AA= 1.00
IMAJ:	DD= -3.00	TO BB= 3.00

BRANCH STARTING AT (.0054698) + J(.051752)

42

ENTER GAIN OF INTEREST (GAIN),TOLERANCE (GTOL):>-8.19,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.6  
h = 0 ft

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -8.190

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 8.954

+OR- KTOL = .1093

ROOTS OF INTEREST

X = -.10129E-01  
X = -.10129E-01  
X = -5.3066  
X = -5.3066

Y = .59194E-02  
Y = -.59194E-02  
Y = -5.7410  
Y = 5.7410

REGION OF CALCULATION-REAL: CC= -5.00

TO AA= 1.00

IMAJ: DD= -3.00

TO BB= 3.00

BRANCH STARTING AT (-.0098176) + J(.0091793)

42

ENTER GAIN OF INTEREST (GAIN),TOLERANCE (GTOL):>-4.89,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.6  
h = 20K ft

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -4.890

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 6.439

+OR- KTOL = .1317

ROOTS OF INTEREST

X = -.24374E-02  
X = -.24374E-02  
X = -3.5940  
X = -3.5940

Y = .22060E-01  
Y = -.22060E-01  
Y = 4.0887  
Y = -4.0887

REGION OF CALCULATION-REAL: CC= -5.00

TO AA= 1.00

IMAJ: DD= -3.00

TO BB= 3.00

BRANCH STARTING AT (-.0019634) + J(.024173)

42

ENTER GAIN OF INTEREST (GAIN),TOLERANCE (GTOL):>-3.21,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.6  
h = 40K ft

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -3.210 +OR- GTOL = .1000  
 SENSITIVITY OF INTEREST (OLK) = 6.425 +OR- KTOL = .2001  
 ROOTS OF INTEREST  
 X = .35518E-02 Y = .36205E-01  
 X = .35518E-02 Y = -.36205E-01  
 X = -3.3303 Y = -2.9933  
 X = -3.3303 Y = 2.9933

REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00  
 IMAJ: DD= -3.00 TO BB= 3.00  
 BRANCH STARTING AT (.0040106) + J(.03834)  
 42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -12.94, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42 M = 0.8  
 h = 0 ft

CODE  
 0-LOCUS PT. 2-ZERO 4-IMAGINARY AXIS  
 1-POLE 3-BREAK PT. 5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -12.94 +OR- GTOL = .1000  
 SENSITIVITY OF INTEREST (OLK) = 13.45 +OR- KTOL = .1039

ROOTS OF INTEREST  
 X = -.33700E-01 Y = 0.  
 X = -.46191E-03 Y = 0.  
 X = -7.9555 Y = 7.4579  
 X = -7.9555 Y = -7.4579

REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00  
 IMAJ: DD= -3.00 TO BB= 3.00  
 BRANCH STARTING AT (-.03328) + J(0.)  
 42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -9.87, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42 M = 1  
 h = 20K ft

CODE  
 0-LOCUS PT. 2-ZERO 4-IMAGINARY AXIS  
 1-POLE 3-BREAK PT. 5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -9.870 +OR- GTOL = .1000  
 SENSITIVITY OF INTEREST (OLK) = 8.994 +OR- KTOL = .9113E-0

ROOTS OF INTEREST  
 X = -.24331E-01 Y = 0.  
 X = .14732E-02 Y = 0.  
 X = -5.0638 Y = -5.3380  
 X = -5.0638 Y = 5.3380

REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00

IMAJ: DD= -3.00 TO BE= 3.00  
 BRANCH STARTING AT (-.02451) + J(0.)  
 42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): >-5.22, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 1.0  
 h = 40K ft

CODE  
 0-LOCUS PT. 2-ZERO 4-IMAGINARY AXIS  
 1-POLE 3-BREAK PT. 5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -5.220 +OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 5.774 +OR- KTOL = .1106

ROOTS OF INTEREST

X = -.39191E-02	Y = .11737E-01
X = -.39191E-02	Y = -.11737E-01
X = -3.1235	Y = -3.9261
X = -3.1235	Y = 3.9261

REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00  
 IMAJ: DD= -3.00 TO BB= 3.00

BRANCH STARTING AT (-.0037698) + J(.012623)  
 42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): >-13.3, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 1.2  
 h = 20K ft

CODE  
 0-LOCUS PT. 2-ZERO 4-IMAGINARY AXIS  
 1-POLE 3-BREAK PT. 5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -13.30 +OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 9.334 +OR- KTOL = .7018E-0

ROOTS OF INTEREST

X = -.33977E-01	Y = 0.
X = .25656E-02	Y = 0.
X = -5.2366	Y = 5.4320
X = -5.2366	Y = -5.4320

REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00  
 IMAJ: DD= -3.00 TO BB= 3.00

BRANCH STARTING AT (-.034739) + J(0.)  
 42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): >-8.24, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 1.4  
 h = 40K ft

CODE  
 0-LOCUS PT. 2-ZERO 4-IMAGINARY AXIS  
 1-POLE 3-BREAK PT. 5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -8.240

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 5.736

+OR- KTOL = .6961E-0

ROOTS OF INTEREST

X = -.30835E-03

Y = 0.

X = -.15795E-01

Y = 0.

X = -3.1342

Y = -4.3811

X = -3.1342

Y = 4.3811

REGION OF CALCULATION-REAL: CC= -5.00

TO AA= 1.00

IMAJ: DD= -3.00

TO BB= 3.00

BRANCH STARTING AT (-.015595) + J(0.)

42

## Method .II

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -3.77, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.3  
h = 0 ft

CODE

0-LOCUS PT.

2-ZERO

4-IMAGINARY AXIS

1-POLE

3-BREAK PT.

5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -3.770

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 5.897

+OR- KTOL = .1564

ROOTS OF INTEREST

X = .25857E-02

Y = .54302E-01

X = .25857E-02

Y = -.54302E-01

X = -3.2390

Y = 3.1384

X = -3.2390

Y = -3.1384

REGION OF CALCULATION-REAL: CC= -5.00

TO AA= 1.00

IMAJ: DD= -3.00

TO BB= 3.00

BRANCH STARTING AT (.0043183) + J(.060757)

42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -3.35, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.4  
h = 20K ft

CODE

0-LOCUS PT.

2-ZERO

4-IMAGINARY AXIS

1-POLE

3-BREAK PT.

5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -3.350

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 6.110

+OR- KTOL = .1824

ROOTS OF INTEREST

X = .36759E-02

Y = .48291E-01

X = .36759E-02

Y = -.48291E-01

X = -3.2338

Y = 3.0047

X = -3.2338

Y = -3.0047

REGION OF CALCULATION-REAL: CC= -5.00

TO AA= 1.00

IMAJ: DD= -3.00

TO BB= 3.00

BRANCH STARTING AT (.0046977) + J(.052395)

42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -2.65, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.5  
h = 40K ft

CODE

0-LOCUS PT.

2-ZERO

4-IMAGINARY AXIS

1-POLE

3-BREAK PT.

5-SENSITIVITY PT.



GAIN OF INTEREST (GAIN) = -2.650 +OR- GTOL = .1000  
SENSITIVITY OF INTEREST (OLK) = 8.829 +OR- KTOL = .3332

ROOTS OF INTEREST

X = .50115E-02	Y = .48984E-01
X = .50115E-02	Y = -.48984E-01
X = -4.4256	Y = -2.3687
X = -4.4256	Y = 2.3687

REGION OF CALCULATION-REAL: CC= -15.0 TO AA= 1.00  
IMAJ: DD= -15.0 TO BB= 15.0  
BRANCH STARTING AT (.0054698) + J(.051752)  
42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -9.42, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42 M = 0.6  
h = 0 ft

CODE

0-LOCUS PT.	2-ZERO	4-IMAGINARY AXIS
1-POLE	3-BREAK PT.	5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -9.420 +OR- GTOL = .1000  
SENSITIVITY OF INTEREST (OLK) = 10.30 +OR- KTOL = .1093

ROOTS OF INTEREST

X = -.10164E-01	Y = .54389E-02
X = -.10164E-01	Y = -.54389E-02
X = -5.9789	Y = 5.2494
X = -5.9789	Y = -5.2494

REGION OF CALCULATION-REAL: CC= -15.0 TO AA= 1.00  
IMAJ: DD= -15.0 TO BB= 15.0  
BRANCH STARTING AT (-.0098176) + J(.0091793)  
42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -5.58, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42 M = 0.6  
h = 20K ft

CODE

0-LOCUS PT.	2-ZERO	4-IMAGINARY AXIS
1-POLE	3-BREAK PT.	5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -5.580 +OR- GTOL = .1000  
SENSITIVITY OF INTEREST (OLK) = 7.347 +OR- KTOL = .1317

ROOTS OF INTEREST

X = -.24935E-02	Y = .21802E-01
X = -.24935E-02	Y = -.21802E-01
X = -4.0482	Y = 3.7316
X = -4.0482	Y = -3.7316

REGION OF CALCULATION-REAL: CC= -15.0 TO AA= 1.00  
IMAJ: DD= -15.0 TO BB= 15.0

GAIN OF INTEREST (GAIN) = -10.45

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 9.523

+OR- KTOL = .9113E-0

1

ROOTS OF INTEREST

X = -.24323E-01	Y = 0.
X = .14568E-02	Y = 0.
X = -5.3280	Y = -5.1358
X = -5.3280	Y = 5.1358

REGION OF CALCULATION-REAL: CC= -5.00

TO AA= 1.00

IMAJ: DD= -3.00

TO BB= 3.00

BRANCH STARTING AT (-.02451) + J(0.)

42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -5.92, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 1.0

h = 40K ft

CODE

0-LOCUS PT.

2-ZERO

4-IMAGINARY AXIS

1-POLE

3-BREAK PT.

5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -5.920

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 6.549

+OR- KTOL = .1106

ROOTS OF INTEREST

X = -.39368E-02	Y = .11629E-01
X = -.39368E-02	Y = -.11629E-01
X = -3.5106	Y = -3.6391
X = -3.5106	Y = 3.6391

REGION OF CALCULATION-REAL: CC= -5.00

TO AA= 1.00

IMAJ: DD= -3.00

TO BB= 3.00

BRANCH STARTING AT (-.0037698) + J(.012623)

42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -12.92, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 1.2

H = 20K ft

CODE

0-LOCUS PT.

2-ZERO

4-IMAGINARY AXIS

1-POLE

3-BREAK PT.

5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -12.92

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 9.068

+OR- KTOL = .7018E-0

1

ROOTS OF INTEREST

X = -.33995E-01	Y = 0.
X = .25804E-02	Y = 0.
X = -5.1033	Y = 5.5252
X = -5.1033	Y = -5.5252

BRANCH STARTING AT  $(-.0019634) + j(.024173)$   
42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL):  $-3.18, .1$

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

$M = 0.6$   
 $h = 40K \text{ ft}$

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) =  $-3.180$

+OR- GTOL =  $.1000$

SENSITIVITY OF INTEREST (OLK) =  $6.365$

+OR- KTOL =  $.2001$

ROOTS OF INTEREST

$X = .35558E-02$   
 $X = .35558E-02$   
 $X = -3.3003$   
 $X = -3.3003$

$Y = .36223E-01$   
 $Y = -.36223E-01$   
 $Y = -3.0230$   
 $Y = 3.0230$

REGION OF CALCULATION-REAL: CC=  $-15.0$

TO AA=  $1.00$

IMAJ: DD=  $-15.0$

TO BB=  $15.0$

BRANCH STARTING AT  $(.0040106) + j(.03834)$   
42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL):  $-12.95, .1$

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

$M = 0.8$   
 $h = 0 \text{ ft}$

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) =  $-12.95$

+OR- GTOL =  $.1000$

SENSITIVITY OF INTEREST (OLK) =  $13.46$

+OR- KTOL =  $.1039$

ROOTS OF INTEREST

$X = -.33701E-01$   
 $X = -.46181E-03$   
 $X = -7.9607$   
 $X = -7.9607$

$Y = 0.$   
 $Y = 0.$   
 $Y = 7.4540$   
 $Y = -7.4540$

REGION OF CALCULATION-REAL: CC=  $-15.0$

TO AA=  $1.00$

IMAJ: DD=  $-15.0$

TO BB=  $15.0$

BRANCH STARTING AT  $(-.03328) + j(0.)$   
42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL):  $-10.64, .1$

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

$M = 1.0$   
 $h = 20K \text{ ft}$

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) =  $-10.64$

+OR- GTOL =  $.1000$

SENSITIVITY OF INTEREST (OLK) = 9.696 +OR- KTOL = .9113E-0

1

ROOTS OF INTEREST

X = -.24320E-01	Y = 0.
X = .14515E-02	Y = 0.
X = -5.4146	Y = -5.0648
X = -5.4146	Y = 5.0648

REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00

IMAJ: DD= -3.00 TO BB= 3.00

BRANCH STARTING AT (-.02451) + J(0.)

42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL):>-6.01,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 1.0

h = 40K ft

CODE

0-LOCUS PT.

2-ZERO

4-IMAGINARY AXIS

1-POLE

3-BREAK PT.

5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -6.010

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 6.646

+OR- KTOL = .1106

ROOTS OF INTEREST

X = -.39390E-02	Y = .11616E-01
X = -.39390E-02	Y = -.11616E-01
X = -3.5604	Y = -3.5975
X = -3.5604	Y = 3.5975

REGION OF CALCULATION-REAL: CC= -5.00

TO AA= 1.00

IMAJ: DD= -3.00

TO BB= 3.00

BRANCH STARTING AT (-.0037698) + J(.012623)

42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL):>-13.11,.1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 1.2

h = 20K ft

CODE

0-LOCUS PT.

2-ZERO

4-IMAGINARY AXIS

1-POLE

3-BREAK PT.

5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -13.11

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 9.201

+OR- KTOL = .7018E-0

1

ROOTS OF INTEREST

X = -.33986E-01	Y = 0.
X = .25730E-02	Y = 0.
X = -5.1699	Y = 5.4792
X = -5.1699	Y = -5.4792

REGION OF CALCULATION-REAL: CC= -5.00

TO AA= 1.00

IMAJ: DD= -3.00

TO BB= 3.00

BRANCH STARTING AT  $(-.034739) + j(0.)$   
42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL):  $-9.46, .1$

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

$M = 1.4$   
 $h = 40K \text{ ft}$

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) =  $-9.460$

+OR- GTOL =  $.1000$

SENSITIVITY OF INTEREST (OLK) =  $6.585$

+OR- KTOL =  $.6961E-0$

1

ROOTS OF INTEREST

$X = -.30155E-03$

$Y = 0.$

$X = -.15790E-01$

$Y = 0.$

$X = -3.5589$

$Y = 4.1167$

$X = -3.5589$

$Y = -4.1167$

REGION OF CALCULATION-REAL: CC=  $-5.00$

TO AA=  $1.00$

IMAJ: DD=  $-3.00$

TO BB=  $3.00$

BRANCH STARTING AT  $(-.0155395) + j(0.)$

42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -3.49, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.3  
h = 0 ft

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -3.490

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 5.459

+OR- KTOL = .1564

ROOTS OF INTEREST

X = .26956E-02  
X = .26956E-02  
X = -3.0201  
X = -3.0201

Y = .54711E-01  
Y = -.54711E-01  
Y = 3.3035  
Y = -3.3035

REGION OF CALCULATION-REAL: CC= -5.00

TO AA= 1.00

IMAJ: DD= -3.00

TO BB= 3.00

BRANCH STARTING AT (.0043183) + J(.060757)

42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -3.06, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.4  
h = 20K ft

CODE

0-LOCUS PT.  
1-POLE

2-ZERO  
3-BREAK PT.

4-IMAGINARY AXIS  
5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -3.060

+OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 5.581

+OR- KTOL = .1824

ROOTS OF INTEREST

X = .37562E-02  
X = .37562E-02  
X = -2.9694  
X = -2.9694

Y = .48607E-01  
Y = -.48607E-01  
Y = -3.2268  
Y = 3.2268

REGION OF CALCULATION-REAL: CC= -5.00

TO AA= 1.00

IMAJ: DD= -3.00

TO BB= 3.00

BRANCH STARTING AT (.0046977) + J(.052395)

42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -2.37, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

CODE



IMAJ: DD= -3.00 TO BB= 3.00  
 BRANCH STARTING AT (-.0019634) + J(.024173)  
 42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -2.88, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.6  
 h = 40K ft

CODE  
 0-LOCUS PT. 2-ZERO 4-IMAGINARY AXIS  
 1-POLE 3-BREAK PT. 5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -2.880 +OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 5.764 +OR- KTOL = .2001

ROOTS OF INTEREST

X = .35963E-02	Y = .36407E-01
X = .35963E-02	Y = -.36407E-01
X = -3.0001	Y = -3.2901
X = -3.0001	Y = 3.2901

REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00  
 IMAJ: DD= -3.00 TO BB= 3.00  
 BRANCH STARTING AT (.0040106) + J(.03834)  
 42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -12.66, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 0.8  
 h = 0 ft

CODE  
 0-LOCUS PT. 2-ZERO 4-IMAGINARY AXIS  
 1-POLE 3-BREAK PT. 5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -12.66 +OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 13.16 +OR- KTOL = .1039

ROOTS OF INTEREST

X = -.30701E-01	Y = 0.
X = -.51049E-03	Y = 0.
X = -7.8115	Y = 7.5555
X = -7.8115	Y = -7.5555

REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00  
 IMAJ: DD= -3.00 TO BB= 3.00  
 BRANCH STARTING AT (-.03328) + J(0.)  
 42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -10.45  
 .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 1.0  
 h = 20K ft

CODE  
 0-LOCUS PT. 2-ZERO 4-IMAGINARY AXIS



REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00  
 IMAJ: DD= -3.00 TO BB= 3.00  
 BRANCH STARTING AT (-.034739) + J(0.)  
 42

ENTER GAIN OF INTEREST (GAIN), TOLERANCE (GTOL): -9.27, .1

OPEN-LOOP (OLTF) ROOT LOCUS USING OPTION 42

M = 1.4  
 h = 40K ft

CODE  
 0-LOCUS PT. 2-ZERO 4-IMAGINARY AXIS  
 1-POLE 3-BREAK PT. 5-SENSITIVITY PT.

GAIN OF INTEREST (GAIN) = -9.270 +OR- GTOL = .1000

SENSITIVITY OF INTEREST (OLK) = 6.453 +OR- KTOL = .6961E-0

1

ROOTS OF INTEREST

X = -.30269E-03	Y = 0.
X = -.15816E-01	Y = 0.
X = -3.4927	Y = 4.1618
X = -3.4927	Y = -4.1618

REGION OF CALCULATION-REAL: CC= -5.00 TO AA= 1.00  
 IMAJ: DD= -3.00 TO BB= 3.00  
 BRANCH STARTING AT (-.015595) + J(0.)

## Appendix F

### Least Squares Curve Fitting

To find the best equation for the gain schedule of  $K_q$ , the least squares method was used. A curve must be fit to the data points in Fig 10. The basic system equation is the starting point:

$$Ax = b$$

Premultiplying by  $A^T$  gives:

$$A^T Ax = A^T b$$

Solving for  $x$ , the result is:

$$x = (A^T A)^{-1} A^T b$$

In this case, the vector  $x$  contains the coefficients for the equation of the curve which fits the points. The work done in Method III, which was discussed in the section, "Selection of Gains," will be shown.

For this case, the points in Fig 10 were divided into two groups and a curve was fit through each group. The lower six points were in one group and the upper five were in another. For group 1:

$$A = \begin{bmatrix} 1 & Q_1 \\ 1 & Q_2 \\ 1 & Q_3 \\ 1 & Q_4 \\ 1 & Q_5 \\ 1 & Q_6 \end{bmatrix} = \begin{bmatrix} 1 & 69 \\ 1 & 99 \\ 1 & 109 \\ 1 & 134 \\ 1 & 246 \\ 1 & 275 \end{bmatrix}$$

and

$$b = \begin{bmatrix} K_{q_1} \\ K_{q_2} \\ K_{q_3} \\ K_{q_4} \\ K_{q_5} \\ K_{q_6} \end{bmatrix} = \begin{bmatrix} 2.08 \\ 3.00 \\ 3.18 \\ 3.65 \\ 5.25 \\ 5.98 \end{bmatrix}$$

In this case

$$\begin{aligned} x &= \left[ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 69 & 99 & 109 & 134 & 246 & 275 \end{bmatrix} \begin{bmatrix} 1 & 69 \\ 1 & 99 \\ 1 & 109 \\ 1 & 134 \\ 1 & 246 \\ 1 & 275 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 69 & 99 & 109 & 134 & 246 & 275 \end{bmatrix} \begin{bmatrix} 2.08 \\ 3.00 \\ 3.18 \\ 3.65 \\ 5.25 \\ 5.98 \end{bmatrix} \\ &= \begin{bmatrix} 1.174 \\ 0.01727 \end{bmatrix} \end{aligned}$$

The vector  $x$  has given us the coefficients for the equation:

$$K_q = C + DQ$$

We found that:

$$K_q = 1.174 + 0.01727Q$$

This same procedure is repeated for the upper five points. Now:

$$\begin{aligned}
 & \times \\
 & = \left[ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 535 & 539 & 682 & 951 & 982 \end{bmatrix} \begin{bmatrix} 1 & 535 \\ 1 & 539 \\ 1 & 682 \\ 1 & 951 \\ 1 & 982 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 535 & 539 & 682 & 951 & 982 \end{bmatrix} \begin{bmatrix} 8.55 \\ 10.25 \\ 10.08 \\ 12.21 \\ 13.45 \end{bmatrix} \\
 & = \begin{bmatrix} 4.839 \\ 0.008226 \end{bmatrix}
 \end{aligned}$$

This results in the equation:

$$K_q = 4.839 + 0.008226Q$$

At the intersection of the two lines, one finds the Q at which the gain schedule will change from one equation to the other. Setting

$$1.174 + 0.01727Q = 4.839 + 0.008226Q$$

one finds that the crossover Q is 405 lb/ft<sup>2</sup>.

Therefore, the gain schedule for K<sub>q</sub> is:

For Q less than or equal to 405 lb/ft<sup>2</sup>

$$K_q = 1.174 + 0.01727Q$$

And for Q greater than 405 lb/ft<sup>2</sup>

$$K_q = 4.839 + 0.008226Q$$

## VITA

Holly Lynn Emrick was born on 29 March 1958 at Wheelus AB, Tripoli, Libya. She graduated from Wheat Ridge High School in Wheat Ridge, Colorado in 1976. She then entered the United States Air Force Academy and upon graduation in May 1980, she received the degree of Bachelor of Science in Aeronautical Engineering and a commission in the USAF. She worked as a stability and control engineer at the Flight Dynamics Laboratory at Wright-Patterson AFB from June 1980 until entering the School of Engineering, Air Force Institute of Technology, in June 1982.

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFIT/GAE/AA/83S-3	2. GOVT ACCESSION NO. N6-AG-2814	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) DESIGN OF LONGITUDINAL CONTROL LAWS FOR THE X-29A BACKUP ANALOG FLIGHT CONTROL SYSTEM		5. TYPE OF REPORT & PERIOD COVERED
7. AUTHOR(s) Lt Holly L. Emrick		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Insittute of Technology (AFIT/EN) Wright-Patterson AFB, OH 45433		8. CONTRACT OR GRANT NUMBER(s)
11. CONTROLLING OFFICE NAME AND ADDRESS Flight Dynamics Laboratory (ASD/AFWAL) Wright-Patterson AFB, OH 45433		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE September 1983
		13. NUMBER OF PAGES 145
		15. SECURITY CLASS. (of this report)
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)  Flight Control System Feedback Control Static Instability		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  This report describes the design of longitudinal control laws for the X-29A backup analog flight control system. Classical feedback design is used for the linear, rigid body model. The resulting control laws provide Level 1 handling qualities performance.		

END

FILMED

3-84

DTIC